# MODELING NATURAL GAS NETWORKS FOR PLANNING AND SCHEDULING

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**Abstract.** In this paper we describe a modular, constrained based model for a gas transmission network, developed to answer questions concerning the supply, demand and transportation in the context of an optimization approach. The model approximates the non-linear relationship for pipes and compressor stations. The goal of the model is not the on-line control of the network but a precise enough estimation of its transport capacity to be used in a wider logistic model. The precision can be adjusted depending on the requirements.

**Key Words.** Constraints, Logistics, Gas Transmission, Pipes, Compressor Stations

## 1. INTRODUCTION

Gas transmission pipeline networks are non-linear, very complex, great scale distributed systems. They can have thousands of pipes, production, storing and distribution centers, compressor stations and many other physical devices as valves and regulators. Transmission networks work at very high pressures and, as the gas flow through the pipes, energy is lost, so compressor stations are needed to give the gas the necessary energy to move it through long distances. A compressor station may have a number of compressor units with different characteristics but all of them with a highly non-linear behavior; moreover, every unit may independently be switched on or off to the network. The gas flow has one way stretches and others where the direction may reverse depending of the network conditions. Furthermore, the number of compressors, sinks and sources may vary with time, yielding a hybrid system.

The main concern when operating and planning a gas pipeline system is minimizing cost while maximizing throughput. In the literature on this topic, it is always considered a number of sources, sinks and compressor stations joined by pipes. Most papers on the subject try to minimize the energy consumption of the compressor stations working in a stationary mode.

In the pioneering paper [1] some rules of thumb are given on how to operate the gas transmission network with low energy consumption. Since then, there have not been many contributions due to the complexity of the topic. The gas flow control is done in a non-automatic way. Based on experience and simulation results, the network operators modify the flow and pressure in the network, turn on/off individual compressor units and fix their set points to satisfy customer demand with minimal operating cost [5].

The paper [6] may be considered the first serious attempt in developing an optimization algorithm for fuel cost minimization in steady state gas transmission networks. They use dynamic programming to solve problems with only one "leg" and compressor stations in series.

Though there has been other attempts, no considerable advance was achieved in the way this problem is dealt with till the work [3]. They use a dynamic simulator and a successive quadratic programming optimizer to compute the optimal compressor operating policies so that their energy consumption is minimized, while keeping the system in a "safe region". They consider constant gas flow and pressure at sources and compressor stations with identical compressor units with ideal efficiency. They approximate the nonconvex non-linear envelope that defines the operation of every compressor unit by a linear, convex envelope.

A more complex and realistic model is considered in [7]. They also consider the problem of minimizing the fuel cost of the compressor stations under steady state conditions. However, they consider the possible reversibility of gas flow in some network stretches and the number of units operating within each compressor station. They give two model relaxations, one in the compressor envelope and another in the fuel cost function, and derive a lower bounding scheme.

The non-linear and complex relationships between flows and pressures of pipes and compressors make difficult their modeling for planning and scheduling purposes. In this paper we describe a modular, constrained based model for a Spanish gas transmission network, developed to answer questions concerning the supply, demand and transportation in the context of an optimization approach. The model approximates the non-linear relationship for pipes and compressor stations, and we can adjust its precision in terms of the requirements.

The goal of the model is not the on-line control of the network but a precise enough estimation of its transport capacity to be used in a wider logistic model. In this setting, we identify a medium-term logistic model (intended for a time period of a month) which abstracts the physical details of the network and allows us to take high level decisions as how much gas buy from what importer in order to sign the best contract in advance considering the data obtained from our model.

In addition, we identify a short-term logistic model (intended for diary operations) that we use to check whether the high level decisions are compatible with the physical details of the network. This entails to check whether the diary operations can be carried out throughout the month, day by day, with the limits imposed by the medium-term logistic model.

The paper is organized as follows. Section 2 discusses the problem domain. Section 3 presents the pipe model we have developed and Section 4 the compressor unit and stations models. Finally the conclusions are given in Section 5.

## 2. PROBLEM DOMAIN

Gas transportation networks consist of pipes, compressor stations and many other devices, such as valves and regulators. The network transports gas coming from gasifier plants and natural sources. Gasifier plants collect Liquefied Natural Gas (LNG) from different wells using a fleet of ships. The gas is delivered to customers with a pressure within a specified interval. Fig. 1 gives a schematic view of the network considered in this paper.

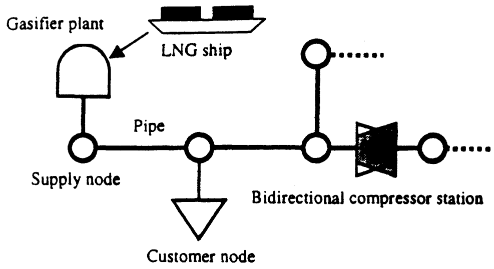


Fig. 1. Schema of a branch of the natural gas network considered in this paper

### 2.1 Pipes

A pipe is the most important component of the network. They have lengths that range from a few tens of kilometers to more than 100 km, with diameters ranging from 300 to 12,000 mm. Although unidirectional flow is usually assumed when modeling the flow of gas through a pipe, in the context of a scheduling scheme it is necessary to model the bidirectionality. The steady state pipe flow equation takes the following form:

$$p_2^2 = \alpha p_1^2 - \beta Q^2 \tag{1}$$

where $p_1$ and $p_2$ are pressure at the end nodes of the pipe, $Q$ is mass flow rate through the pipe and $\alpha$, $\beta$ are quantities depending on the pipe physical attributes: gas compressibility factor, gas specific gravity, average temperature (assumed constant), frictional factor, length, inside diameter of pipe and the inclination angle of the pipeline segment [4]. Every source and delivery node has a pressure range $[p_{min}, p_{max}]$ where must be contained any valid pressure for this node.

### 2.2 Compressor stations

These elements give the flow the required energy to reach the delivery nodes with the required pressure and the demanded quantity. A compressor station consists of several compressor units in parallel. Each unit could be turned on or off, and its behavior is nonlinear. For the same reason that pipes, some compressors must be modeled as bidirectional elements.

The power consumption, assuming that the compressors work adiabatically, when compressing a gas from the suction pressure $p_s$, to the discharge pressure $p_d$ at a volumetric flow rate $q$ is:

$$HP = C_1\, q\, H \tag{2}$$

where $C_l$ depends on the gas pumped and $H$ is the adiabatic head, that is given by

$$H = \frac{C_2}{\kappa}\left[\left(\frac{p_d}{p_s}\right)^{\kappa} - 1\right] \tag{3}$$

where $C_2$ depends on the gas pumped and on the suction temperature, that we consider constant, and $\kappa$ depends on the gas pumped. Moreover, the volumetric flow rate and the suction pressure are related with the mass flow rate through the equation:

$$Q = \frac{q\ p_s}{Z_s} \tag{4}$$

and where $Z_s$ is given approximately by

$$Z_s = 1 - \frac{p_s}{390} \tag{5}$$

Fig. 2 shows a plot relating $H$ and $q$ for a typical compressor unit.

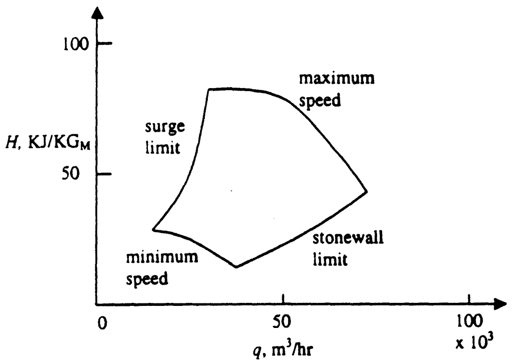


Fig. 2. Envelope for a typical compressor unit

### 2.3 Constraint model of the system

The planning and scheduling system for gas supply uses an optimization model (mixed-integer/ constraint programming) and consists of a set of interconnected modules. Each module implements the behavior of a physical device of the gas transmission network. The behavior is modeled as a global constraint over hydraulic variables of the device. In a constraint model the relationships between variables must be provided for each possible direction. That is, if $p_1$ and $p_2$ are the gas pressures at both ends of a pipe and the mass flow rate $Q$, the constraint model for the pipe $C(p_1, p_2, Q)$ ensures that the 3 values $(p_1, p_2, Q)$ are always compatible with the hydraulic behavior of the pipe. We are mainly interested in flow capacity of the pipes and compressors in order to define the specific activities to be performed over a time scale of days.

## 3. PIPES

This section deals with implementing gas pipe behavior from the model stated above. A gas pipe is defined by pressures at both ends and the gas flow. In general, pressure decays with flow direction, but it can also augment because of a decrement in potential energy. Our model is posed as a piecewise linear function that relates pressures at both ends ($p_1$ and $p_2$) with the gas flow ($Q_t$), and that approximates the nonlinear function:

$$p_1(p_2, Q_t) = a(Q_t) \cdot p_2 + b(Q_t) + c(Q_t) \cdot Q_t \tag{6}$$

where $a(Q_t)$, $b(Q_t)$, and $c(Q_t)$ coefficients are defined in terms of $Q_t$. Coefficients of a possible pipe is defined as shown in Table 1, which gathers coefficients for each piece.

Table 1. Coefficients for a possible pipe

| $Q_t$ | $a(Q_t)$ | $b(Q_t)$ | $c(Q_t)$ |
|---|---|---|---|
| $[-400,-300]$ | 0.9889 | 0.0189 | -0.0039 |
| $[-300,-200]$ | 0.9947 | 0.0144 | -0.0028 |
| $[-200,0]$ | 0.9997 | 0.0391 | -0.0011 |
| $[0,200]$ | 1.0003 | -0.0391 | -0.0011 |
| $[200,300]$ | 1.0053 | -0.0145 | -0.0028 |
| $[300,400]$ | 1.0112 | -0.0191 | -0.0039 |

### 3.1 Constraint Modeling of Pipes

The pipe constraint model is connected to the transmission network (nodes, compressor stations and gasifiers) by means of real variables representing the three magnitudes defining a pipe (pressures and flow). The domain of the flow variable is real whereas pressure variables domains are positive real. The function $p_1(p_2, Q_t)$ can be expressed as:

$$p_1(p_2, Q_t) = \begin{cases} a_0 \cdot p_2 + b_0 + c_0 \cdot Q_t, if\ Q_t \in [Q_{t_1}(0), Q_{t_2}(0)] \\ a_1 \cdot p_2 + b_1 + c_1 \cdot Q_t, if\ Q_t \in [Q_{t_1}(1), Q_{t_2}(1)] \\ \vdots \\ a_{n-1} \cdot p_2 + b_{n-1} + c_{n-1} \cdot Q_t, if\ Q_t \in [Q_{t_1}(n-1), Q_{t_2}(n-1)] \end{cases}$$

where:

$$a(Q_t) = a_i,\ Q_t \in [Q_{t_1}(i), Q_{t_2}(i)]$$

$$b(Q_t) = b_i,\ Q_t \in [Q_{t_1}(i), Q_{t_2}(i)]$$

$$c(Q_t) = c_i,\ Q_t \in [Q_{t_1}(i), Q_{t_2}(i)]$$

This function can be expressed by the following constant (discontinuous) piecewise function, which defines planes for each flow piece:

$$p_1(p_2, Q_t) = \sum_{i=0}^{n-1} (a(Q_t) \cdot p_2 + b(Q_t) + c(Q_t) \cdot Q_t) \cdot s_i(Q_t) \tag{7}$$

where $s_i(Q_t) = \begin{cases} 1, if\ Q_t \in [Q_{t_1}(i), Q_{t_2}(i)] \\ 0,\ e.o.c. \end{cases}$

and $a_i$, $b_i$ and $c_i$ coefficients come from Table 1 (an example with n=6).

This function involves nonlinear terms which are products of a constant, a positive real or real variable, and a binary variable. They are modeled with an approach similar to [2] but enhanced by taking advantage of the positive domain. Further enhancements include adding the following domain constraints:

- $p_1 \in [Minpi(t), Maxpi(t)]$
- $p_2 \in [Minpf(t), Maxpf(t)]$
- $Q_t \in [MinVarxfi(t), MaxVarxfi(t)]$

where:

- *Minpi(t)*, *Minpf(t)*: Minimum pressure at end 1 (resp. 2) of the pipe, which is determined by physical parameters of the network and is obtained from Table 1.
- *Maxpi(t)*, *Maxpf(t)*: Maximum pressure at end 1 (resp. 2) of the pipe, which is determined by physical parameters of the network and is obtained from Table 1.
- *MinVarxfi(t)*, *MaxVarxfi(t)*: Minimum (maximum, resp.) flow through pipe in piece *t*, which is determined by physical parameters of the network and is obtained from Table 1.

### 3.2 Input Parameters

Discretization of the function relating $Q_t$, $p_1$ and $p_2$ is defined by a set of tuples:

$$Q = \{< t, Q_{t_1}, Q_{t_2}, a, b, c >\},$$

which states that, for a piece $t$ and a normal condition flow $Q_t \in [Q_{t_1}, Q_{t_2}]$, then:

$$p_1(p_2, Q_t) = a \cdot p_2 + b + c \cdot Q_t \tag{8}$$

Each tuple in $Q$ belongs to the table schema:

$< \underline{t} : text, \underline{Q_{t_1}} : float, Q_{t_2} : float, a : float, b : float, c : float >$

where underscored parameters forms the primary key, *float* denotes a real field, and *text* denotes a textual field. A candidate key is $< t, Q_{t_2} >$. The following are further integrity constraints.

a) Domain constraint: $Q_{t_1} \le Q_{t_2}$.

b) Functional dependencies:

- $t, Q_{t_1} \rightarrow a, b, c$
- $t, Q_{t_1} \rightarrow t, Q_{t_2}$

c) Uniqueness:

- $\forall u, v \in R:$

$u = < t, Q_{t_1}(i), Q_{t_2}(i), a(i), b(i), c(i) >,$
$v = < t, Q_{t_1}(j), Q_{t_2}(j), a(j), b(j), c(j) >,$
$u \ne v \Rightarrow$

$$[Q_{t_1}(i), Q_{t_2}(i)] \cap [Q_{t_1}(j), Q_{t_2}(j)] = \varnothing$$

### 3.3 Validation

Results from the implementation above have been validated by examining the relation $p_1 = f(p_2, Q_t)$. We have generated test vectors covering the plane $p_2 - Q_t$ to get $p_1$ values. The set $\{< p_2, Q_t >\}$ represents a dense point grid in this plane. The selection of this set is based on a plane covering by regular intervals of $p_2$ and $Q_t$. For each element in this set a run is executed so that actual results can be matched with run results. We have used Matlab to perform the validation. Fig. 3 shows the validation test results, and Fig. 4 shows a cut in the plane $p_2 = 30$ to illustrate the nonlinear behavior of the relation.

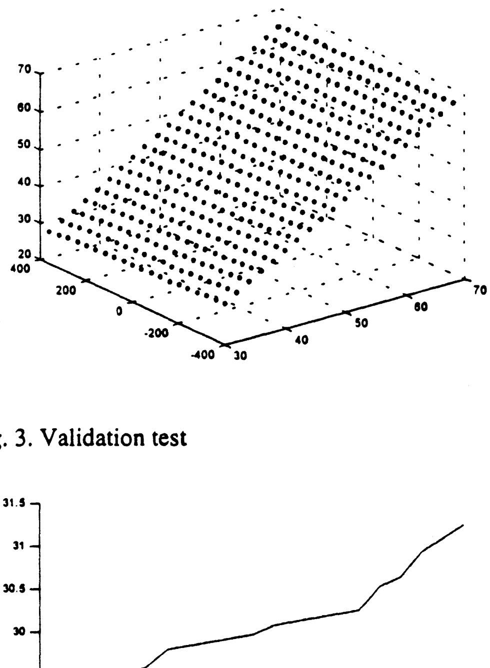


Fig. 3. Validation test

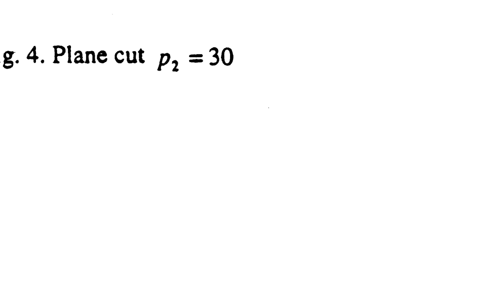


Fig. 4. Plane cut $p_2 = 30$

## 4. COMPRESSOR STATIONS

The compressor station model relates pressures at both ends with the mass flow rate ($Q_t$). A compressor station can be disabled (equal pressures at both ends) or enabled (several compressor units can be enabled). In addition, it can work in two possible directions; an inversion in the direction means the inversion of the roles of pressures at both ends. The least pressure of both ends is known as suction pressure ($p_s$) whereas the upper pressure is known as discharge pressure ($p_d$). A compressor raises suction pressure to discharge pressure through the use of compressor units. Discharge pressure is a function of gas flow in normal conditions and suction pressure:

$$p_d = f(Q_t, p_s)$$

This function has been translated to the discontinuous domain so that we have defined maximum and minimum values for the discharge pressure ($p_{dmax}(i,j)$ and $p_{dmin}(i,j)$, resp.) for each surface defined by the intervals $[Q_t(i), Q_t(i+1))$ and $[p_s(j), p_s(j+1))$, where $Q_t(i)$ and $p_s(i)$ are the values partitioning the axis $Q_t$ and $p_s$, resp. Therefore, the new expression for $p_d$ is:

$$p_{dmin}(Q_t, p_s) \le p_d(Q_t, p_s) \le p_{dmax}(Q_t, p_s), \qquad (9)$$

where:

$$p_{dmin}(Q_t, p_s) = p_{dmin}(i,j), Q_t \in [Q_t(i), Q_t(i+1)), p_s \in [p_s(j), p_s(j+1))$$
$$p_{dmax}(Q_t, p_s) = p_{dmax}(i,j), Q_t \in [Q_t(i), Q_t(i+1)), p_s \in [p_s(j), p_s(j+1))$$

Figures 5 and 6 show minimum and maximum discharge pressure (vertical axis) related with suction pressure (axis pointing to the left) and gas flow (axis pointing to the rignt).

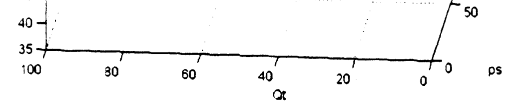


Fig. 5. Minimum discharge pressure

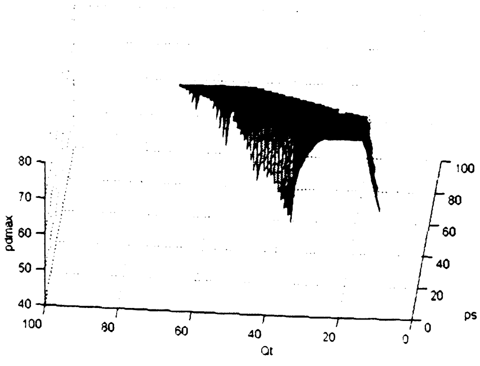


Fig. 6. Maximum discharge pressure

The specification of the modeled compressor station is as follows, which involves the use of $n$ compressor units enabled.

- Compressor station disabled: $p_s = p_d, \forall Q_t$
- Compressor station enabled, for each turbocompressor:

$$p_{dmin}(Q_t/n, p_s) \le p_d(Q_t, p_s) \le p_{dmax}(Q_t/n, p_s) \qquad (10)$$

This means that when the compressor station is enabled, the gas flow is divided into the enabled turbocompressors. The relation $p_d(Q_t, p_s)$ refers to the total gas flow, whereas $p_{dmin}(Q_t, p_s)$ and $p_{dmax}(Q_t, p_s)$ are normalized to one compressor.

### 4.1 Constraint Modeling of Compressor Stations

As in the pipe model, the compressor station model is connected to the transmission network by means of real variables representing pressures and flow with the same domains.

When the compressor station is disabled, there is no constraint on gas flow, which is modeled with a binary variable stating whether the compressor station is enabled or disabled. Nonlinear terms caused by the binary variable are treated as in [2] with enhancements considering the domains. The same binary variable controls the connection of constraints on pressures at both ends.

In order to express the relation between flow, suction pressure, and discharge pressure with respect to the number of compressor units enabled, we use a variable $qf$ which represent gas flow through each compressor unit. So:

$$qi = qf + \sum_{i=1}^{num\_units-1} i \cdot unit(i) \cdot qf$$

Where $unit(i)$ is a binary variable which represents enabling $i+1$ turbocompressors, $num\_units$ is the total number of compressors, and $qi$ is the total flow through the compressor station.

The specification of the modeled compressor station assumes an absolute value for the flow and we use the variable $qt = |qf|$, which adds another source of nonlinearity.

In addition, since this specification assumes a unique flow direction, the constraint model has also to embody the switching of pressure roles. A new binary variable *positive* is used to denote the gas flow sign (direction). A positive flow says that pressure in end $i$, $pi$, is the suction pressure, and the pressure in end $f$, $pf$, is the discharge pressure. So:

$$positive = 1 \Rightarrow pi = ps, pf = pd$$

$$positive = 0 \Rightarrow pi = pd, pf = ps$$

where the variables $ps$ and $pd$ have been added to represent suction and discharge pressures, so that we have represented the behavior of a station compressor with one turbocompressor enabled as $pd = f(qt, ps)$.

In order to implement the relation among $Q_t$, $p_s$ and $p_d$, we express equation 10 as:

$$p_d(Q_t, p_s) \le \sum_j \sum_i p_{dmax}(i,j) \cdot \delta(i,j)$$

$$p_d(Q_t, p_s) \ge \sum_j \sum_i p_{dmin}(i,j) \cdot \delta(i,j)$$

where $\delta(i,j)$ are binary variables which take value 1 when $Q_t \in [Q_t(i), Q_t(i+1))$ and $p_s \in [p_s(j), p_s(j+1))$. These last expressions relate $i$ and $j$ values with those of $Q_t$ and $p_s$, or alternatively:

$$Q_t \in [Q_t(i), Q_t(i+1)), \quad p_s \in [p_s(j), p_s(j+1)) \quad \Leftrightarrow \quad \delta(i,j) = 1$$

In order to enhance performance we include domain and redundant constraints:

a) Domain constraints:

- $qi \in [-MaxVarxfci..MaxVarxfci]$
- $qt \in [0..Maxqt]$
- $qf \in [-Maxqt..Maxqt]$
- $p_s \in [psmin..psmax]$
- $p_d \in [0..MaxPressure]$

Where:

- *MaxPressure*: Maximum pressure at each end, which is determined by physical parameters of the network.
- *MaxVarxfci*: Maximum flow through a compressor station, which is determined by its physical parameters.
- *Maxqt*: Maximum flow at each turbocompressor as absolute value, which is determined by its physical parameters.

$$Maxqt = max(\left\{\begin{array}{l} qti, qtf: \\ < psi, psf, qti, qtf, pdmin, pdmax > \in Q \end{array}\right\})$$

where $Q$ is the relation presented in the next section.

- *psmin, psmax*: Minimum (resp. Maximum) suction prèssion for each turbocompressor, which is determined by its physical parameters. Operationally, they are obtained as:

$$psmin = min(\left\{\begin{array}{l} psi: \\ < psi, psf, qti, qtf, pdmin, pdmax > \in Q \end{array}\right\})$$

$$psmax = max(\left\{\begin{array}{l} psf: \\ < psi, psf, qti, qtf, pdmin, pdmax > \in Q \end{array}\right\})$$

b) Redundant constraints:

- Gas flow limitation:

$$Q_t \in [Q_t(i), Q_t(i+1)), p_s \in [p_s(j), p_s(j+1)) \Rightarrow Q_{tmin}(j) \le Q_t \le Q_{tmax}(j)$$

- Alternatives limitation:

$$Q_t \ge Q_{tmax}(j), p_s \in [p_s(j), p_s(j+1)) \Rightarrow \delta(i,j) = 0, \quad \forall i > k : qt(j) = max(Q_{tmax}(k))$$

$$Q_t \le Q_{tmin}(i), p_s \in [p_s(j), p_s(j+1)) \Rightarrow \delta(i,j) = 0, \quad \forall i < k : qt(j) = min(Q_{tmin}(k))$$

where $Q_{tmin}(j)$ and $Q_{tmax}(j)$ are the minimum and maximum values of each interval for $p_s$, that is:

$$Q_{tmin}(j) \le Q_t \le Q_{tmax}(j), p_s \in [p_s(j), p_s(j+1))$$

Increasing indexes ($i$ and $j$) correspond to increasing flows and pressures, which is a relevant relation for input parameters.

### 4.2 Input Parameters

Discretization of the function relating $Q_t$, $p_s$ and $p_d$ is defined by a set of tuples:

$Q = \{< Q_t(i), Q_t(i+1), p_s(j), p_s(j+1), p_{dmin}(i,j), p_{dmax}(i,j) >\}$,

which states that, for a normal condition flow $Q_t \in [Q_t(i), Q_t(i+1))$ and a suction pressure $p_s \in [p_s(j), p_s(j+1))$, then:

$$p_{dmin}(i,j) \leq p_d(Q_t, p_s) \leq p_{dmax}(i,j)$$

Each tuple above belongs to the following table schema:

$$< \underline{Q_{ti}} : float, Q_{tf} : float, \underline{p_{si}} : float+, p_{sf} : float+,$$
$$p_{dmin} : float+, p_{dmax} : float+ >$$

where underscored parameters forms the primary key. A candidate key is $< Q_{tf}, p_{sf} >$. The following are further integrity constraints.

a) Domain constraints:

- $Q_{ti} \leq Q_{tf}$
- $p_{si} \leq p_{sf}$
- $p_{dmin} \leq p_{dmax}$

b) Functional dependencies:

- $t, Q_{t_1} \rightarrow t, Q_{t_2}, a, b, c$
- $Q_{tf}, p_{sf} \rightarrow Q_{ti}, p_{si}$

### 4.3 Validation

Results validation comes from two complementary ways. First, assuming that the function $p_d = f(Q_t, p_s)$ is correctly implemented, the selection of representative cases for verifying the connection of turbocompressors to the compressor station. Second, the verification of correctness of $p_d = f(Q_t, p_s)$, as in Section 3.3.

#### 4.3.1 Verifying Turbocompressor Connection

We have generated test vectors for verifying:

- Compressor station enabling implies pressures related by $p_d = f(Q_t, p_s)$.
- Compressor station disabling implies pressures equated.
- Flow direction correctly relates suction and discharge pressures.
- Total flow through the compressor is the sum of the flows through each enabled turbocompressor.

#### 4.3.2 ~~Verifying Turbocompressor Connection~~

Results from the implementation above have been validated by verifying the equation 1. We have generated test vectors covering the plane $Q_t - p_s$ under two assumptions: maximization and minimization of the cost function $p_d$, so that we can get the maximum and minimum values for $p_d$ (valid range). These test vectors are sketched in Table 2.

Table 2. Test Vectors Description

| Test Vector | Derivations | Situation |
|---|---|---|
| $\{< Q_t, p_s >\}$ max( $p_d$ ) | $\{p_{dmax} = f(Q_t, p_s)\}$ | Compressor station with a turbocompressor enabled. Maximum discharge pressure. |
| $\{< Q_t, p_s >\}$ min( $p_d$ ) | $\{p_{dmin} = f(Q_t, p_s)\}$ | Compressor station with a turbocompressor enabled. Minimum discharge pressure. |

The set $\{< Q_t, p_s >\}$ represents a dense point grid in the plane $Q_t - p_s$. The selection of this set is based on a plane covering by regular intervals of $Q_t$ and $p_s$. For each element in this set with cardinality 30,000, a run is executed so that actual results can be matched with run results. We have used Matlab to perform the validation. Figures 7 and 8 show the validation test results.

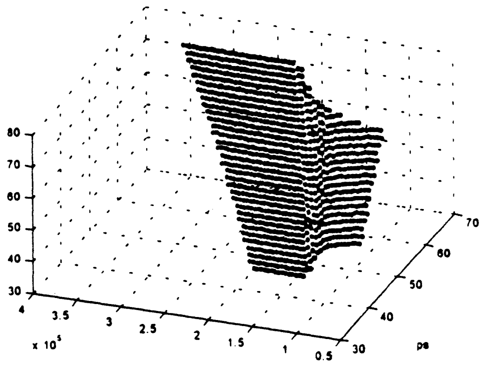


Fig. 7. Validation tests for $p_{dmin}$

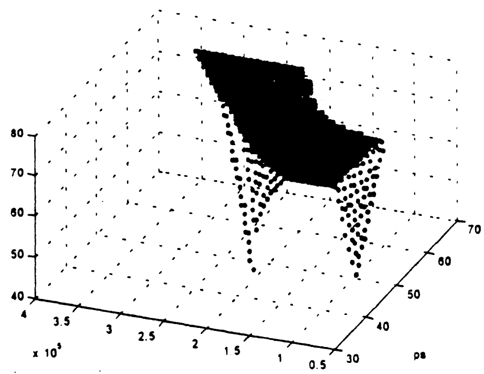


Fig. 8. Validation tests for $p_{dmax}$

## 5. CONCLUSIONS

We have developed a modular constraint based model for a gas transmission network. This model can be used in planning and scheduling optimization systems. The model approximates the whole complexity of the nonlinear relationship for pipes and compressor stations. The model can be easily adjusted to get the required precision to be used in the planning system.

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