A new algorithm to compute low T-Transitive approximation of a fuzzy relation preserving symmetry. Comparisons with the T-transitive closure.

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Abstract. It is given a new algorithm to compute a lower T-transitive approximation of a fuzzy relation that preserves symmetry. Given a reflexive and symmetric fuzzy relation, the new algorithm computes a T-indistinguishability that is contained in the fuzzy relation. It has been developed a C++ program that generates random symmetric fuzzy relations or random symmetric and reflexive fuzzy relations and computes their T-transitive closure and the new low T-transitive approximation. Average distances of the fuzzy relation with the T-transitive closure are similar than the average distances with the low T-transitive approximation.

1 Introduction

Fuzzy relations have many applications to make fuzzy inference in many branches of Artificial Intelligence with uncertainty, imprecision or lack of knowledge. Reflexive and T-transitive fuzzy relation (called T-preorders, for any continuous t-norm T) make Tarski consequences when using the composite rule of inference, obtaining all the consequences of a few premises in just one S-T-composition. Reflexive symmetric and T-transitive fuzzy relations (called T-indistinguishabilities) have been very useful in many classification and clustering methods, allowing to represent the knowledge to distinguish objects.

A new method to T-transitivize fuzzy relations [Garmendia & Salvador; 2000] can be used to measure of T-transitivity of fuzzy relations and to build T-transitive low approximations of a given fuzzy relations. That algorithm preserves all the diagonal values, so it preserves the α-reflexivity, however it doesn't preserve the symmetry property, so we have developed a different version of the algorithm that keeps the symmetry property.

Fuzzy relations on a finite set can also represent labeled directed graphs. The T-transitive closure generalize the transitive closure of a directed graph, and lower T-transitive approximations are T-transitive subgraphs. Symmetric fuzzy relations can represent non directed graphs, where a generalized transitive property could be studied or inferred.

The new algorithm is implemented in a C++ program that generate random symmetric fuzzy relations or random reflexive and symmetric fuzzy relations of a given dimension and computes their Min-transitive closure, Prod- transitive closure and W-transitive closure, and compares them with their Min-transitive, Prod-transitive and W-transitive low symmetric approximations using the new proposed algorithm.

It is computed the measure of low T-transitivity of fuzzy relations measuring the difference between the transitive low approximations and the original fuzzy relation, using several distances as the absolute value of the difference, euclidean distances or normalized distances. Those distances are also measured between the same random fuzzy relations and their T-transitive closures, resulting to be higher than the average distances with the T-transitive low approximations for all dimensions computed.

2 Preliminaries

2.1 The importance of the transitivity property

The T-transitive property is held by T-indistinguishabities and T-preorders, and it is important when making fuzzy inference to have *Tarski* consequences. The similarities and T-indistinguishabilities generalize the classical equivalence relations, and are useful to classify or to make fuzzy partitions of a set. T-indistinguishability relations generalize the classical equivalence relations and they are useful to define degrees of 'similarities' or generalized distances.

Even though not all the fuzzy inference in control needs transitivity, it looks important to know whether the fuzzy relation is T-transitive in order to make fuzzy inference, and if a relation is not T-Transitive it is possible to find another T-transitive fuzzy relation as close as possible with the initial fuzzy relation.

2.2 Transitive closure

The T-transitive closure R^T of a fuzzy relation R is the lowest relation that contains R and is T-transitive. There are many proposed algorithms to compute the T-transitive closure [Naessens, De Meyer, De Baets; 2002].

An algorithm used to compute the transitive closure is the following:

1) $R' = R \cup_{Max} (R o_{Sup\text{-}T} R)$
2) If $R' \neq R$ then $R := R'$ and go back to 1), otherwise stop and $R^T := R'$.

2.3 A new T-transitivization algorithm

At *'On a new method to T-transitivize fuzzy relations'* [Garmendia & Salvador; 2000] it is proposed a new algorithm to compute low T-transitive approximations of fuzzy relations, obtaining a fuzzy T-transitive relation 'as close as possible' from the initial fuzzy relation. If the initial relation is T-transitive then it is equal to the T-transitivized relation.

The transitivized relation keeps important properties as the μ-T-conditionality property and reflexivity that also preserves the transitive closure, but it also keeps some more properties as the invariance of the relation degree of every element with himself (or diagonal), and so it preserves α-reflexivity. The transitivity closure does not preserve α-reflexivity, but preserves symmetry.

2.4 Previous concepts

Let $E = \{a_1, ..., a_n\}$ be a finite set.

A fuzzy relation $R: E\times E \to [0, 1]$ is a T-indistinguishability when it is reflexive, symmetric and T-transitive.

A T-indistinguishability is called a similarity when T is the minimum t-norm.

Definition 1*:* Let T be a triangular t-norm [Schweizer & Sklar; 1983]. A fuzzy relation $R: E\times E \to [0, 1]$ is **T-transitive** if $T(R(a,b), R(b,c)) \leq R(a,c)$ for all a, b, c in E.

Given a fuzzy relation R it is called element $a_{i,j}$ to the relation degree in [0, 1] between the elements a_i and a_j in E. So $a_{i,j} = R(a_i, a_j)$.

Definition 2*:* An element $a_{i,j}$ is called **T-transitive element** if $T(a_{i,k}, a_{k,j}) \leq a_{ij}$ for all k from 1 to n.

Algorithm*:* The proposed algorithm transform a fuzzy relation R^0 into another T-transitive relation R_T contained in R^0 in n^2-1 steps. In each step can be reduced some degrees so $R = R^0 \supseteq R^1 \supseteq ... \supseteq R^m \supseteq ... \supseteq R^{n^2-1} = R_T$.

The idea of this method is to get profit of the fact that each step makes sure that an element $a_{i,j}$ will be T-transitive for all further steps, and so it will be T-transitive in the final relation R_T. In summary, each step m+1 T-transitivize an element $a_{i,j}^m$ in R^m reducing other elements $a_{i,k}^m$ or $a_{k,j}^m$, when it is necessary, resulting that $a_{i,j}^r$ is T-transitive in R^r for all $r \geq m$. To achieve this, it is important to choose in each step the minimum non T-transitivized element as the candidate to transitivize (reducing other elements). When choosing to transitivizate the minimum $a_{i,j}^m$ in R^m it is sure that $a_{i,j}^m = a_{i,j}^r$ for all $r \geq m$ (it will not change in further steps), because the reduction of other elements will not make it intransitive anymore and because $a_{i,j}^m$ is lower or equal further transitivized elements, it will not cause intransitivity and it will not be reduced.

Let τ be a set of pairs (i, j) where i, j are integers from 1 to n.

Definition 3*:* τ^m is a subset of τ defined by:

1) $\tau^0 = \emptyset$

2) $\tau^{m+1} = \tau^m \cup (i, j)$ if $a_{i,j}{}^m$ is the element in R^m chosen to be T-transitivized in the m+1 step.

So τ^m is the set of pairs (i, j) corresponding the T-transitivized elements in R^m and $(\tau^m)'$ is the set of n^2-m pairs (i, j) corresponding the not yet transitivized elements.

Building R^{m+1} from R^m: Let $a_{i,j}{}^m$ be the element in R^m that is going to be transitivized at step m+1 ($a_{i,j}{}^m = \text{Min}\{a_{v,w}{}^m$ such that $(v, w) \in (\tau^m)'\}$).

It is defined $a_{r,s}{}^{m+1}$ as

$$\begin{cases} J^T(a^m_{s,j}, a^m_{i,j}) & \text{if } r=i,\ T(a^m_{r,s}, a^m_{s,j}) > a^m_{i,j} \text{ and } a^m_{i,s} \le a^m_{s,j} \\ J^T(a^m_{i,r}, a^m_{i,j}) & \text{if } s=j,\ T(a^m_{i,r}, a^m_{r,s}) > a^m_{i,j} \text{ and } a^m_{i,r} \ge a^m_{r,s} \\ a^m_{r,s} & \text{otherwise} \end{cases} \quad (1)$$

where J^T is the residual operator of the t-norm T, defined by $J^T(x, y) = \sup\{z / T(x, z) \le y\}$.

If $T(a_{i,k}{}^m, a_{k,j}{}^m) > a_{ij}{}^m$ for some k, either $a_{i,k}{}^m$ or $a_{k,j}{}^m$ will reduce its degree (it could be chosen the minimum of both) to achieve that $T(a_{i,k}{}^{m+1}, a_{k,j}{}^{m+1}) \le a_{ij}{}^{m+1} = a_{ij}{}^m$.

When choosing the minimum between $a_{i,k}{}^m$ and $a_{k,j}{}^m$ to reduce, if it is chosen the minimum one, the difference between R^m and R^{m+1} is lower, so if $a_{i,k}{}^m \le a_{k,j}{}^m$ then $a_{i,k}{}^{m+1} = J^T(a_{k,j}{}^m, a_{ij}{}^m)$ and if $a_{i,k}{}^m > a_{k,j}{}^m$ then $a_{k,j}{}^{m+1} = J^T(a_{i,k}{}^m, a_{ij}{}^m)$. The degree of the rest of elements remains invariant ($a_{r,s}{}^{m+1} = a_{r,s}{}^m$).

3 A new algorithm to compute low T-Transitive approximation of a fuzzy relation preserving symmetry

Algorithm 2.3 can be used to compute low T-transitive approximations of any fuzzy relations. However, the algorithm can be modified to take profit of the knowledge that the input is going to be a symmetric fuzzy relation

The idea is that when a relation degree $a_{i,j}$ is T-transitivised, we can use the calculations to T-transitivized the symmetric degree $a_{j,i}$ at the same time. So the new algorithm will need half of the steps.

The final algorithm that preserves symmetry is similar to 2.3, but computing $a_{r,s}{}^{m+1}$ at the same time than $a_{s,r}{}^{m+1}$

Let E be a set of n elements and let $R^0 : E \times E \to [0,1]$ be a symmetric fuzzy relation.

Algorithm:

The proposed algorithm transform a fuzzy relation R^0 into another T-transitive relation R_T contained in R^0 in $[n^2/2]$ steps. In each step can be reduced some degrees so $R = R^0 \supseteq R^1 \supseteq \ldots \supseteq R^m \supseteq \ldots \supseteq R^{\left[\frac{n^2}{2}\right]} = R_T$.

Let τ be a set of pairs (i, j) where i, j are integers from 1 to n.

1) $\tau^0=\varnothing$

2) $\tau^{m+1} = \tau^m \cup (i, j) \cup (j, i)$ if $a_{i,j}{}^m$ and is the element in R^m chosen to be T-transitivized at step m+1.

Building R^{m+1} from R^m: Let $a_{i,j}{}^m$ be the element in R^m that is going to be transitivized at step m+1 ($a_{i,j}{}^m = \mathrm{Min}\{a_{v,w}{}^m$ such that $(v, w) \in (\tau^m)'\}$).

It is defined $a_{r,s}{}^{m+1} := a_{s,r}{}^{m+1} :=$

$$\begin{cases} J^T(a^m_{s,j}, a^m_{i,j}) & \text{if } r=i,\ T(a^m_{r,s}, a^m_{s,j}) > a^m_{i,j} \text{ and } a^m_{i,s} \le a^m_{s,j} \\ J^T(a^m_{i,r}, a^m_{i,j}) & \text{if } s=j,\ T(a^m_{i,r}, a^m_{r,s}) > a^m_{i,j} \text{ and } a^m_{i,r} \ge a^m_{r,s} \\ a^m_{r,s} & \text{otherwise} \end{cases}$$

where J^T is the residual operator of the t-norm T, defined by $J^T(x, y) = \sup\{z / T(x, z) \le y\}$.

Example 3.1

Let R be a symmetric fuzzy relation on a set $E = \{a_1, a_2, a_3\}$ defined by the matrix

$$R^0 = \begin{pmatrix} 0,4 & 1 & 0,7 \\ 1 & 0,3 & 0,4 \\ 0,7 & 0,4 & 0,2 \end{pmatrix}$$

To compute the low Min-transitive approximation, the first step is to Min-transitivize the lower relation degree, which is $R(a_3, a_3) = a_{3,3} = 0,2$ using the residuated operator of the Min t-norm on values $a_{3,1}$, $a_{1,3}$ and $a_{3,2}$, $a_{2,3}$, so

$$R^1 = \begin{pmatrix} 0,4 & 1 & 0,2 \\ 1 & 0,3 & 0,2 \\ 0,2 & 0,2 & 0,2 \end{pmatrix}$$

As $a_{3,1}$ and $a_{3,2}$ are Min-transitive (and then their symmetric values), no values are reduced in the next two steps, and $R_2=R_3=R_1$.

The lower non Min-transivized value is $a_{2,2} = 0,3$, that is not Min-transitive. Then

$$R^4 = \begin{pmatrix} 0,4 & 0,3 & 0,2 \\ 0,3 & 0,3 & 0,2 \\ 0,2 & 0,2 & 0,2 \end{pmatrix}$$

$R_T = R^4$ is a low Min-transitive approximation of R

The Min-transitive closure of R is $R^T = \begin{pmatrix} 1 & 1 & 0,7 \\ 1 & 1 & 0,7 \\ 0,7 & 0,7 & 0,7 \end{pmatrix}$, which does not preserve the diagonal values.

4 The program

The most important continuous t-norms that generalize the AND logical values are the Minimum, Product, and the Lukasiewicz t-norm, $W(x, y) = \max\{0, x+y-1\}$.

4.1 Program Description

It has been developed a program in C++ that generates a random symmetric fuzzy relation (shown at the top of the figure) or a random reflexive and symmetric fuzzy relations and computes the Min-transitive closure, Prod-transitive closure and W-transitive closure, measuring the absolute value distance and euclidean distance with the initially generated fuzzy relation. It also computes the Min-transitive, Prod-transitive and W-transitive low approximations (second row of relations in the figure 1), and also measures their distances with the same original fuzzy relation.

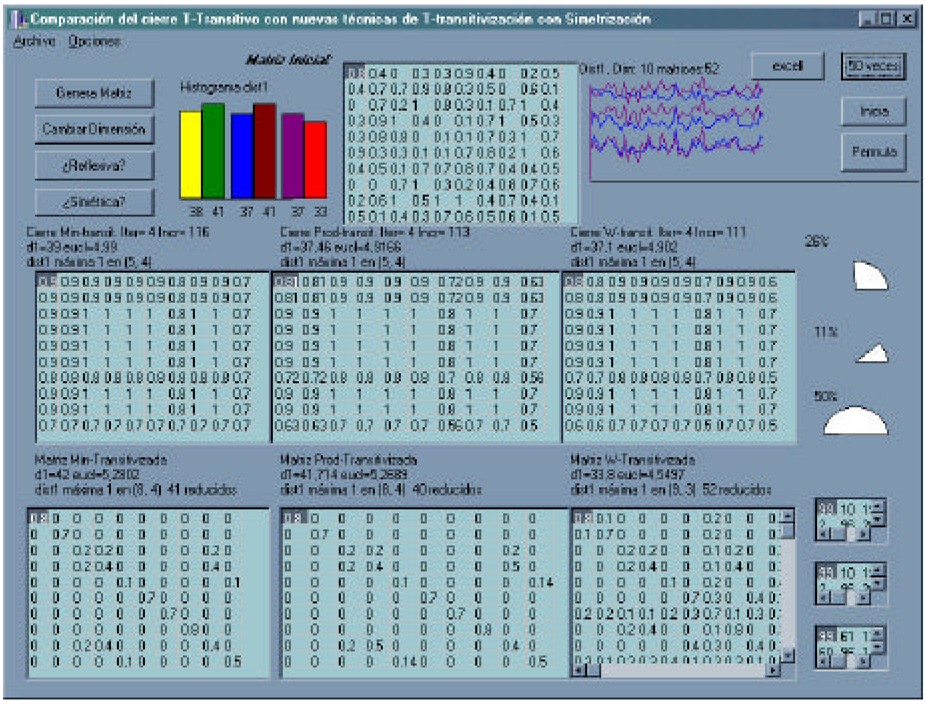

Fig. 1. General front-end of the program.

As an example, the program generates the following random symmetric fuzzy relation:

0.8	0.4	0	0.3	0.3	0.9	0.4	0	0.2	0.5
0.4	0.7	0.7	0.9	0.8	0.3	0.5	0	0.6	0.1
0	0.7	0.2	1	0.8	0.3	0.1	0.7	1	0.4
0.3	0.9	1	0.4	0	0.1	0.7	1	0.5	0.3
0.3	0.8	0.8	0	0.1	0.1	0.7	0.3	1	0.7
0.9	0.3	0.3	0.1	0.1	0.7	0.8	0.2	1	0.6
0.4	0.5	0.1	0.7	0.7	0.8	0.7	0.4	0.4	0.5
0	0	0.7	1	0.3	0.2	0.4	0.8	0.7	0.6
0.2	0.6	1	0.5	1	1	0.4	0.7	0.4	0.1
0.5	0.1	0.4	0.3	0.7	0.6	0.5	0.6	0.1	0.5

Fig. 2. Example of generated symmetric random fuzzy relation.

The program computes the Min-transitive closure, Prod-transitive closure and W-transitive closure measuring the absolute value distance and euclidean distance with the initial fuzzy relation:

Cierre Min-transit. Iter= 4 Incr= 116
d1=39 eucl=4,99
dist1 máxima 1 en (5, 4)

0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.9	0.9	0.7
0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.9	0.9	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7

Cierre Prod-transit. Iter= 4 Incr= 113
d1=37,46 eucl=4,9166
dist1 máxima 1 en (5, 4)

0.81	0.81	0.9	0.9	0.9	0.9	0.72	0.9	0.9	0.63
0.81	0.81	0.9	0.9	0.9	0.9	0.72	0.9	0.9	0.63
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.72	0.72	0.8	0.8	0.8	0.8	0.7	0.8	0.8	0.56
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.63	0.63	0.7	0.7	0.7	0.7	0.56	0.7	0.7	0.5

Cierre W-transit. Iter= 4 Incr= 111
d1=37,1 eucl=4,902
dist1 máxima 1 en (5, 4)

0.8	0.8	0.9	0.9	0.9	0.9	0.7	0.9	0.9	0.6
0.8	0.8	0.9	0.9	0.9	0.9	0.7	0.9	0.9	0.6
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.7	0.7	0.8	0.8	0.8	0.8	0.7	0.8	0.8	0.5
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.9	0.9	1	1	1	1	0.8	1	1	0.7
0.6	0.6	0.7	0.7	0.7	0.7	0.5	0.7	0.7	0.5

Fig. 3. Example of Min-Transitive closure, Prod-transitive closure and W-transitive closure of the random fuzzy relation of Fig. 2, measuring the absolute value distance and euclidean distance with the initial fuzzy relation.

It also computes the Min-transitive, Prod-transitive and W-transitive low approximations (second row of relations in the figure) using the algorithm that preserves symmetry, and also measures their distances with the same original fuzzy relation:

Matriz Min-Transitivizada
d1=42 eucl=5,2802
dist1 máxima 1 en (8, 4) 41 reducidos

0.8	0	0	0	0	0	0	0	0	0
0	0.7	0	0	0	0	0	0	0	0
0	0	0.2	0.2	0	0	0	0	0.2	0
0	0	0.2	0.4	0	0	0	0	0.4	0
0	0	0	0	0.1	0	0	0	0	0.1
0	0	0	0	0	0.7	0	0	0	0
0	0	0	0	0	0	0.7	0	0	0
0	0	0	0	0	0	0	0.8	0	0
0	0	0.2	0.4	0	0	0	0	0.4	0
0	0	0	0	0.1	0	0	0	0	0.5

Matriz Prod-Transitivizada
d1=41,714 eucl=5,2689
dist1 máxima 1 en (8, 4) 40 reducidos

0.8	0	0	0	0	0	0	0	0	0
0	0.7	0	0	0	0	0	0	0	0
0	0	0.2	0.2	0	0	0	0	0.2	0
0	0	0.2	0.4	0	0	0	0	0.5	0
0	0	0	0	0.1	0	0	0	0	0.14
0	0	0	0	0	0.7	0	0	0	0
0	0	0	0	0	0	0.7	0	0	0
0	0	0	0	0	0	0	0.8	0	0
0	0	0.2	0.5	0	0	0	0	0.4	0
0	0	0	0	0.14	0	0	0	0	0.5

Matriz W-Transitivizada
d1=33,8 eucl=4,5497
dist1 máxima 1 en (9, 3) 52 reducidos

0.8	0.1	0	0	0	0	0.2	0	0	[illegible]
0.1	0.7	0	0	0	0	0.2	0	0	[illegible]
0	0	0.2	0.2	0	0	0.1	0.2	0	[illegible]
0	0	0.2	0.4	0	0	0.1	0.4	0	[illegible]
0	0	0	0	0.1	0	0.2	0	0	[illegible]
0	0	0	0	0	0.7	0.3	0	0.4	[illegible]
0.2	0.2	0.1	0.1	0.2	0.3	0.7	0.1	0.3	[illegible]
0	0	0.2	0.4	0	0	0.1	0.8	0	[illegible]
0	0	0	0	0	0.4	0.3	0	0.4	[illegible]
0.3	0.1	0.3	0.3	0.4	0.1	0.3	0.3	0.1	[illegible]

Fig. 4. Example of Min-transitive, Prod-transitive and W-transitive symmetric low approximation of the random fuzzy relation of Fig. 2, measuring the absolute value distance and euclidean distance with the initial fuzzy relation

When choosing to generate reflexive and symmetric random fuzzy relations their computed T-transitive closures will be generated T-indistinguishabilities.

The histogram shows the absolute value distance of the last random generated fuzzy relation with the (in this order from the left to the right) Min-transitive closure, the Min-transitive low approximation, the Prod-transitive closure, the Prod-transitive low approximation, the W-transitive closure and the W-transitive low approximation. The graphic at the right of the picture compares the absolute value distances of both T-transitivization methods for the t-norms (in this order, from the upper to the lower graphs) minimum, product and Lukasiewicz for the last hundred of generated random symmetric fuzzy relations.

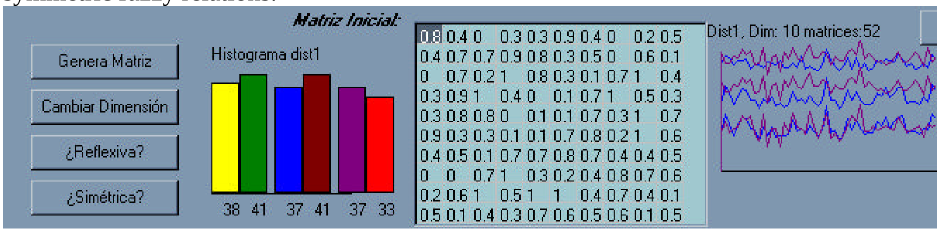

Fig. 8. The histogram shows the absolute value distance of the last random generated fuzzy relation with the Min-transitive closure, the Min-transitive low approximation, the Prod-transitive closure, the Prod- transitive low approximation, the W-transitive closure and the W-transitive low approximation. The graph at the right of the picture compares the absolute value distances of both T-transitivization methods for the t-norms minimum, product and Lukasiewicz for the last hundred of random fuzzy relations.

The dimension can be changed. The results for the relation of example 3.1 are in the following figure:

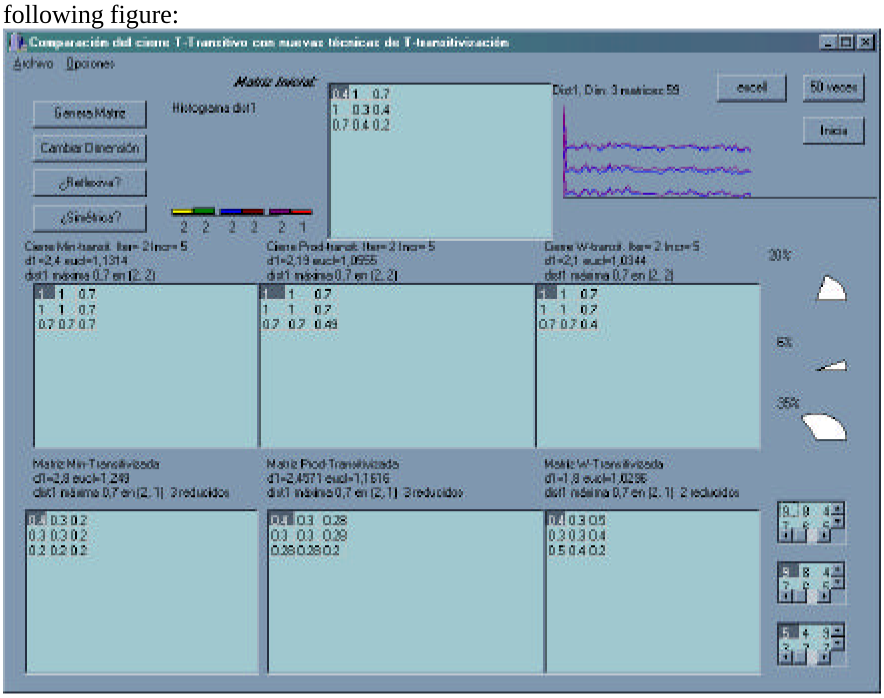

Fig. 9. Program output for example 3.1

The program has been scheduled to generate one hundred of random fuzzy relations for each dimension from two to one hundred. The average distances for each dimension have been saved in an Excel document.

5 Comparing low symmetric T-transitive approximations with T-transitive closures of random reflexive and symmetric fuzzy relation.

It has been run the program one hundred times for each dimension from two to one hundred, it is, the program has generated 9900 random fuzzy reflexive and symmetric relations, computing their T-transitive closures and their T-transitivized relations for different t-norms, and computing their average distance of absolute value and euclidean for each dimension.

The function in the graphic below represents, for each dimension, the average absolute value distance with their W-transitive closure (the line of higher distances) and the W-transitivized relation. The aspect of the results could change when using other distances, but it is got the same looking for the three t-norms used.

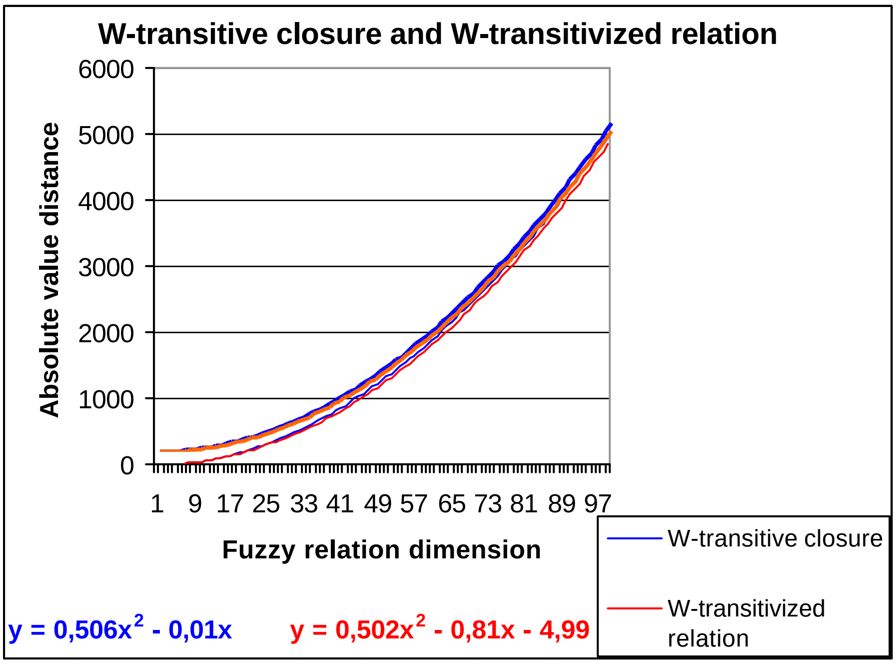

Fig. 9. Average of the absolute value distances of 100 random reflexive and symmetric fuzzy relations with their W-transitive closure and W-transitive low approximation for each dimension from two to one hundred.

Table 2: Interpolation function of the average absolute value distance and euclidean distance of the T-transitive closure and T- transitive low approximation of one hundred random fuzzy relations for each dimension from two to one hundred.

Absolute value distances	**Min**	**Prod**	**W**
Transitive Closure	$y=0,5x^2+1,2x-16,3$	$y=0,59x^2-1,27x+5,9$	$y=0,506x^2-0.01x$
Transitivized relation	$y=0,46x^2-1,27x+5,9$	$y=0,50x^2-0,83x+2,5$	$y=0,502x^2-0,8x-4,9$
Euclidean distances	**Min**	**Prod**	**W**
Transitive Closure	y=0,61x-0,42	y=0,61x-0,63	y=0,61x-0,68
Transitivized relation	y=0,56x-0,76	y=0,56x-0,77	y=0,56x-1,19

6 Results analysis

After generating 100 random fuzzy relations for each dimensions from 2 to 100, and compute their average distance with the T-transitive closure and with the T-transitivized relation, we have seen for any distance, for any t-norm and for any dimension that the T-transitive low approximation is similar to the initial relations than the T-transitive closure.

7 Conclusions

The T-transitivization algorithm that keeps symmetry, applies to reflexive and symmetric fuzzy relations, computes T-transitive low approximations with similar distances than the T-transitive closure for any dimension and any t-norm. They are also different, because computes T-transitive relations contained in the initial relation.

The T-transitive closure is uniquely defined, however we can find several maximal T-transitive relations contained in the initial relation.

It is proven [Garmendia & Salvador; 2000] that the T-transitivization algorithm keeps the reflexivity and α-reflexivity. The new algorithm version also preserves symmetry, so produce T-indistinguishabilities from reflexive and symmetric relations. However the T-transitive closure keeps reflexivity, but not α-reflexivity.

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