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Measuring the specificity of fuzzy sets on infinite domains

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Abstract. A new way for measuring the specificity of fuzzy sets on infinite domains is given. The new expression is defined using t-norms, negations and the Choquet integral. It is also proved that the new expression that satisfies the axioms of measure of specificity. Some examples are provided.

1 Introduction

The concept of specificity is understood as the amount of information contained in a fuzzy set by giving a degree of 'containing just one element'. This is strongly related with the inverse of the cardinality of a set.

If one element of a set must be chosen, and we have a fuzzy set with a degree of satisfaction of each element, it is desirable to have a singleton or a high specificity fuzzy set to make an election with tranquillity.

Some previous works study the measures of specificity of fuzzy sets on discrete domains [9]. The measures of specificity on infinite domains deserve a deeper study.

Garmendia [2] uses a general expression for measures of specificity of fuzzy sets on finite domains using t-norms, t-conorms and negations. The general expression also allows generating many measures of specificity using different fuzzy connectives, so it is possible to find the best measure of specificity of fuzzy sets in every environment or logic.

This paper gives a general expression to measure the specificity of fuzzy sets on infinite domains. The expression uses t-norms, negations, fuzzy measures and the Choquet integral. Some properties and also that other known formulas are particular cases of this general expression are shown. It is proved that this general expression verifies the axioms of measures of specificity [7]. The new measures of specificity are potentially useful in many applications.

R. R. Yager [10] proposes a first expression for measuring the specificity of fuzzy sets on infinite domains that a particular case of the general expression given in this paper. Yeager's first example uses a normalized Lebesgue measure and can be written using the new expression. Several examples of measures of specificity of fuzzy sets on the interval [0, 1] are given using several t-norms.

The new expression of measures of specificity of fuzzy sets on infinite domains can be used to generate different formulas of measure of specificity of fuzzy sets for each environment and for each application.

2 Preliminaries

Definition 2.1: A binary operation T: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a **t-norm** [6], if it satisfies the following axioms:

1. $T(1, x) = x$
2. $T(x, y) = T(y, x)$
3. $T(x, T(y, z)) = T(T(x, y), z)$
4. If $x \leq x'$ and $y \leq y'$ then $T(x, y) \leq T(x', y')$.

A binary operation S: $[0, 1] \times [0, 1]$ is a t-conorm if it satisfies 2, 3, 4 and $S(0, x) = x$.

Definition 2.2: A map N: $[0, 1] \rightarrow [0, 1]$ is a **negation** if it satisfies the following conditions:

1. $N(0) = 1, N(1) = 0$
2. N is non increasing

A negation N is strong if $N(N(x)) = x$.

Definition 2.3: A fuzzy set μ on X is **normal** if there exits an element $x_1 \in X$ such that $\mu(x_1) = 1$.

Definition 2.4. Measure of specificity.

Let X be a set and let $[0, 1]^X$ be the class of fuzzy sets on X. A measure of specificity [9] is a function

Sp: $[0, 1]^X \rightarrow [0, 1]$ such that:

1. $Sp(\emptyset) = 0$.
2. $Sp(\mu) = 1$ if and only if μ is a singleton .
3. If μ and η are normal fuzzy sets in X and $\mu \subset \eta$, then $Sp(\mu) \geq Sp(\eta)$.

The first condition assumes that the empty set have minimum specificity. However other not empty fuzzy sets could also have specificity zero.

The second condition imposes that only crisp sets with just one element (singletons) can have specificity one (the maximum specificity).

The third condition requires that the specificity measure of a normal fuzzy set decreases when the membership degrees of its elements are increased.

Definition 2.5. Regular measure of specificity: A measure of specificity Sp is regular [10] if $Sp(X) = 0$.

Definition 2.6. Weak measures of specificity: Let X be a set with elements $\{x_i\}$ and let $[0, 1]^X$ be the class of fuzzy sets of X. A weak measure of specificity Sp [2] is a function Sp: $[0, 1]^X \to [0, 1]$ such that:

1. $Sp(\emptyset) = 0$
2. $Sp(\mu) = 1$ if μ is a singleton ($\mu=\{x_1\}$).
3. If μ and η are normal fuzzy sets in X and $\mu \subset \eta$, then $Sp(\mu) \geq Sp(\eta)$.

The difference between a measure of specificity and a weak measure of specificity lies on axiom 2. Non-singletons fuzzy sets can have maximum weak specificity.

Definition 2.7. Fuzzy measure [3]**:**

Let $\wp$ be a family of subsets of a set X, with $\emptyset$, X $\in \wp$. A mapping M: $\wp \to [0, 1]$ is called a fuzzy measure if:

1) $M(\emptyset) = 0$
2) $M(X) = 1$
3) If A, B $\in \wp$ and $A \subseteq B$ then $M(A) \leq M(B)$

The triple (X, $\wp$, M) is a **fuzzy measure space**.

In [8] only fuzzy measures that verifies the following condition are considered:

4) M(B) = 0 if and only if B is the empty set or B is a singleton.

Note that condition 4 is a technical condition and it is very difficult to translate in natural language.

The measures of specificity are not fuzzy measures because they are not monotonous with respect to the inclusion of fuzzy sets. The following definition of fuzzy $\leq_k$-measure allows using the word 'measure' to compute the specificity of fuzzy sets, because the measures of specificity are fuzzy $\leq_k$-measures.

Definition 2.8. $\leq_k$-measure [7]:

A measure of a characteristic k shown by the elements of a set E is done through a comparative relation like 'x shows the characteristic k less than y shows it' for any x, y in E.

Let's write '$x \leq_k y$' to denote that relation and suppose that $\leq_k$ is a preorder on E.

A function m: E $\to [0, 1]$ is a fuzzy $\leq_k$-measure for E if it satisfies the following conditions:

1. $m(x_0) = 0$ if $x_0 \in E$ is minimal for $\leq_k$.
2. $m(x_1) = 1$ if $x_1 \in E$ is maximal for $\leq_k$.
3. If $x \leq_k y$ then $m(x) \leq m(y)$.

Remarks

1. Fuzzy measures are $\subseteq$-measures (monotonous measures with the inclusion preorder).
2. The entropy measures [4] for fuzzy sets are $\leq_S$-measures, where $\leq_S$ is the sharpened ordering.

3. The measure of specificity [9] represents the idea of measuring how close is a fuzzy set from a singleton. So, a measure of specificity Sp is a fuzzy $\leq_k$-measure where the set E is $[0, 1]^X$; the characteristic k is the specificity of a fuzzy set; x_0 is the empty set (the only minimal set); x_1 is a singleton (the maximal sets are all singleton) and the preorder $\leq_{Sp}$ is defined as $\mu \leq_{Sp} \sigma \Leftrightarrow Sp(\mu) \leq Sp(\sigma)$.

Definition 2.9. Choquet integral [1]:
Let $(X, \wp, M)$ be a fuzzy measure space. Let f: $X \to [0, \infty]$ be a measurable function. The fuzzy integral of f with respect to a fuzzy measure M by the Choquet integral is:

$$(C)\int_X f \cdot dM = (C)\int_X f(w) \cdot dM(w) = \int_0^\infty M(f(x) > \alpha) \cdot d\alpha \qquad \textbf{(1)}$$

The Choquet integral [5] is an extension of the classical Lebesgue integral for non-classical measures, such as fuzzy measures, which are not necessarily additive measures.

3 An expression for measuring the specificity of fuzzy sets under infinite domains.

The axioms of measure of specificity (definition 2.4) and weak measure of specificity (definition 2.6) of fuzzy sets are given. This paper's goal is to provide expressions and formulas that satisfy the previous axiomatic definitions and that be used when it is useful to measure the amount of information contained in a fuzzy set on an infinite domain in order to make a decision.

A general expression for measures of specificity of fuzzy sets on infinite domains using a t-norm, a strong negation and a fuzzy measure is given and it is proved that the new expression satisfies the weak measures of specificity axioms. When the fuzzy measure verifies the condition 4, which is a technical, then the expression satisfies the axioms for measures of specificity.

Let A be a fuzzy set on an infinite universe X and let α_{sup} be the supreme of the membership degrees of A. Let $(X, \wp, M)$ be a fuzzy measure space (definition 2.7), such that the fuzzy measure M verifies that:

4) M(B) = 0 if and only if B is the empty set or B is a singleton.

Let $A_\alpha \in \wp$ be the α-cut level set of A. Let T be a t-norm (definition 2.1) and let N be a strong negation (definition 2.2).

An expression for measuring the specificity of a fuzzy set A on an infinite domain is given as follows:

$$MS(A) = T(\alpha_{sup}, N(\int_0^{\alpha_{up}} M(A_\alpha) \cdot d\alpha)) \qquad \textbf{(2)}$$

where $\int_0^{\alpha_{sup}}$ is a Choquet integral (definition 2.9).

Lemma 3.1:
If A is a normal fuzzy set then:

$$MS(A) = N(\int_0^1 M(A_\alpha) \cdot d\alpha) \tag{3}$$

Proof:

$$MS(A) = T(\alpha_{sup}, N(\int_0^{\alpha_{sup}} M(A_\alpha) \cdot d\alpha)) = T(1, N(\int_0^1 M(A_\alpha) \cdot d\alpha)) = N(\int_0^1 M(A_\alpha) \cdot d\alpha).$$

Note that if A is a classical non empty set then $MS(A) = N(\int_0^1 M(A_\alpha) \cdot d\alpha)$

Lemma 3.2:
If A and B are non empty classical sets and $M(A) \geq M(B)$ then $MS(A) \leq MS(B)$
The proof is trivial from the previous lemma.

Theorem 3.3:
The measure of specificity expression under infinite domains MS verifies the axioms of measures of specificity (definition 2.4).

Proof

Axiom 1: $MS(\varnothing) = T(0, N(\int_0^0 0 \cdot d\alpha)) = T(0, 1) = 0$

Axiom 2: $MS(\{x\}) = T(1, N(\int_0^1 M(A_\alpha) \cdot d\alpha)) = N(\int_0^1 0 \cdot d\alpha) = N(0) = 1$, and

$MS(A) = 1 \Rightarrow \alpha_{sup} = 1$ and $N(\int_0^1 M(A_\alpha) \cdot d\alpha) = 1 \Rightarrow \alpha_{sup} = 1$ and $\int_0^1 M(A_\alpha) \cdot d\alpha = 0$

$\Rightarrow \alpha_{sup} = 1$ and $M(A_\alpha) = 0$, so by applying the condition 4 it is deduced that A is a singleton.

Axiom 3: If A and B are normal fuzzy sets and $A \subseteq B$ then $M(A_\alpha) \leq M(B_\alpha)$ for any α, so:

$MS(A) = N(\int_0^1 M(A_\alpha) \cdot d\alpha) \geq N(\int_0^1 M(B_\alpha) \cdot d\alpha) = MS(B)$. z

Note that if the condition 4 is not imposed to the fuzzy measure M, then MS is a weak measure of specificity (definition 2.6).

Lemma 3.4:
MS is a regular measure of specificity.

Proof:

$$MS(X) = T(1, N(\int_0^1 M(X_\alpha) \cdot d\alpha)) = N(\int_0^1 1 \cdot d\alpha) = N(1) = 0.$$

4 Measure of specificity for fuzzy sets on infinite domains

R. R. Yager [8] gives a first example of measure of specificity for a fuzzy set on an infinite domain. This paper shows that the same example can be written using the new proposed expression, the usual negation and the Łukasiewicz t-norm.

Let X be an infinite set (for example, a real interval). Let A be a fuzzy set on X and let Aα be its α-cut.

R. R. Yager [1998] proposes a **measure of specificity on an infinite domain** given as follows:

$$Sp(A) = \int_0^{\alpha_{max}} F(M(A_\alpha))\, d\alpha \tag{4}$$

where α_{max} is the maximum membership degree of A, M is a measure on X and F is a function F: [0, 1] $\rightarrow$ [0, 1] verifying:

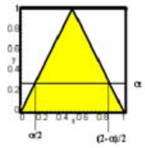
1) F(0) = 1
2) F(1) = 0
3) If x>y then $0 \leq F(x) \leq F(y) \leq 1$

Example 4.1:

Let X be the real interval [0, 1] and let M be the *Lebesgue-Stieltjes* measure defined as M([a, b]) = b–a. Let F be the function F(z) = 1 – z. Let A be the fuzzy set defined by:

$$A(x) = \begin{cases} 2x & 0 \leq x \leq 0.5 \\ -2x+2 & 0.5 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} . \tag{5}$$

The graphical representation of A is the following:



$$\text{Fuzzy set } A(x) = \begin{cases} 2x & 0 \le x \le 0.5 \\ -2x+2 & 0.5 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Fig 1: Fuzzy set A

For any α, $A_\alpha = [\alpha/2, (2-\alpha)/2]$ and $M(A_\alpha) = ((2-\alpha)/2)-(\alpha/2) = 1-\alpha$. As $\alpha_{max}=1$ then:

$$Sp(A) = \int_0^1 F(M(A_\alpha))d\alpha = \int_0^1 (1-(1-\alpha))\, d\alpha = 0.5.$$

Yager gives another new concept for measuring the specificity of fuzzy sets on continuous domains when X is the real interval $[a, b]$ and $F(z)=1-z$:

$$Sp(A) = \int_0^{\alpha_{max}} F(M(A_\alpha))d\alpha = \int_0^{\alpha_{max}} (1-M(A_\alpha))d\alpha = \alpha_{max} - \int_0^{\alpha_{max}} M(A_\alpha)d\alpha$$

If M is the normalized Lebesgue measure $M(B) = \text{Length}(B)/(b-a)$ then

$$Sp(A) = \alpha_{max} - \int_0^{\alpha_{\max}} M(A_\alpha)\, d\alpha = \alpha_{max} - \frac{1}{b-a}\int_0^{\alpha_{max}} \text{Length}(A_\alpha)\, d\alpha.$$

So, the expression $\int_0^{\alpha_{max}} \text{Length}(A_\alpha)d\alpha$ can be interpreted as the area under the fuzzy set A, and the measure of specificity of a fuzzy set A on an interval [a, b] can be given as

$$\alpha_{max} - \frac{\textit{area under } A}{b-a}. \qquad (6)$$

5 The new expression generalises Yeager's measure of specificity of fuzzy sets on infinite domains [8]

It is shown that the previous example 4.1 is a weak measure of specificity (definition 2.6) under an infinite domain when N is the negation $N(x)=1-x$, T is the Łukasiewicz t-norm defined by T(x, y) = max(0, x+y−1), and M is the Lebesgue measure given by the length of an interval. Then

$$MS(A) = \max\left(0, \alpha_{max} + N\left(\int_0^{\alpha_{max}} M(A_\alpha).d\alpha\right) - 1\right)$$

$$= \max\left(0, \alpha_{max} + 1 - \int_0^{\alpha_{max}} M(A_\alpha).d\alpha - 1\right)$$

$$= \max\left(0, \alpha_{max} - \int_0^{\alpha_{max}} M(A_\alpha).d\alpha\right) \text{ (}M(A_\alpha)\text{ is always less or equal than one).}$$

$$= \alpha_{max} - \int_0^{\alpha_{max}} M(A_\alpha).d\alpha = \int_0^{\alpha_{max}} (1 - M(A_\alpha))d\alpha = \int_0^{\alpha_{max}} F(M(A_\alpha))d\alpha = Sp(A).$$

Note: When the measure M is the length of an interval, it does not verify condition 4, hence the new given expression is a weak measure of specificity. For example, if

$$A(x) = \begin{cases} 1 & \textit{if } x = 0, x = 0{,}25, x = 0{,}5, x = 1 \\ 0 & \textit{otherwise} \end{cases}$$

$M(A_1) = 0$ and $M(A_0) = 1$, then $\int_0^1 M(A_\alpha).d\alpha = 0$ and $Sp(A) = 1 - \int_0^1 M(A_\alpha).d\alpha = 1$,

but A is not a singleton.

6 Examples

Example 6.1
To compute a weak measure of specificity of the fuzzy set

$$B(x) = \begin{cases} 0 & 0 \le x \le 0.25 \\ 4x - 1 & 0.25 \le x \le 0.5 \\ -4x + 3 & 0.5 \le x \le 0.75 \\ 0 & 0.75 \le x \le 1 \end{cases} \tag{7}$$

on the real interval [0, 1], it is necessary to compute its α-cut. which is graphically shown in the following figure:

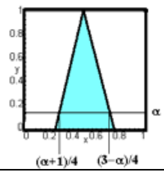


Fig 2: α-cut of B(*x*)

For any α, $B_\alpha = [(\alpha+1)/4, (3-\alpha)/4]$.
If T is the Łukasiewicz t-norm, N(*x*) = 1-*x* and M is the Lebesgue measure then:

$$M(B_\alpha) = \frac{3-\alpha}{4} - \frac{\alpha+1}{4} = \frac{1-\alpha}{2}.$$

As $\alpha_{max} = 1$ it follows that

$$MS(B) = 1 - \int_0^1 M(B_\alpha)d\alpha = 1 - \int_0^1 \frac{1-\alpha}{2} d\alpha = 1 - \frac{1}{4} = \frac{3}{4}.$$

Example 6.2
To compute a weak measure of specificity of the fuzzy set

$$B(x) = \begin{cases} 0 & 0 \le x \le 0.25 \\ 4x-1 & 0.25 \le x \le 0.5 \\ -4x+3 & 0.5 \le x \le 0.75 \\ 0 & 0.75 \le x \le 1 \end{cases} \quad (8)$$

on the real interval [0, 1], it is necessary to compute its α-cut. For any α, $C_\alpha = [\alpha, 1-\alpha]$ and $M(C_\alpha) = 1-\alpha-\alpha = 1-2\alpha$, so $\alpha_{max} = 1/2$.
If N(x) = 1-x and T is the Łukasiewicz t-norm then

$$MS(C) = 1/2 - \int_0^{1/2} M(C_\alpha)d\alpha = 1/2 - \int_0^{1/2} (1-2\alpha).d\alpha = 1/2 - [1/2 - 1/4] = \frac{1}{4}.$$

If T = Prod then

$$MS(C) = Prod(\alpha_{max}, N(\int_0^{1/2} M(C_\alpha)d\alpha)) = 1/2*(1-[1/2-1/4]) = 1/2*(3/4) = 3/8 = 0{,}375.$$

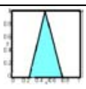
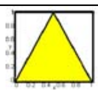
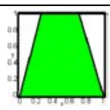
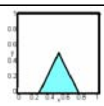
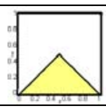
If T = Min then

$$MS(C) = Min(\alpha_{max}, N(\int_0^{1/2} M(C_\alpha)d\alpha)) = Min(1/2, 3/4) = \frac{3}{4} = 0{,}75.$$

Example 6.3

The following table summarises several measures of specificity of five fuzzy sets defined on the unit interval when T is the minimum, Product or Łukasiewicz t-norm, N(x)=1-x and M is the Lebesgue measure.

Table 1: Examples of weak measures of specificity when N(x)=1-x, T = Min, Prod, W, and M is the Lebesgue

X = [0, 1]		T =	Łu-kasiewicz	Product	Minimum
B		$B(x)=\begin{cases} 0 & 0\le x\le 0.25 \\ 4x-1 & 0.25\le x\le 0.5 \\ -4x+3 & 0.5\le x\le 0.75 \\ 0 & 0.75\le x\le 1 \end{cases}$	0.75	0.75	0.75
A		$A(x)=\begin{cases} 2x & 0\le x\le 0.5 \\ -2x+2 & 0.5\le x\le 1 \\ 0 & \text{otherwise} \end{cases}$	0.5	0.5	0.5
E		$E(x)=\begin{cases} 4x & 0\le x\le \frac{1}{4} \\ 1 & \frac{1}{4}\le x\le \frac{3}{4} \\ 4-4x & \frac{3}{4}\le x\le 1 \end{cases}$	0.25	0.25	0.25
D		$D(x)=\begin{cases} 0 & 0\le x\le 0.25 \\ 2x-1/2 & 0.25\le x\le 0.5 \\ 3/2-2x & 0.5\le x\le 0.75 \\ 0 & 0.75\le x\le 1 \end{cases}$	0.375	0.437	0.5
C		$C(x)=\begin{cases} x & 0\le x\le 0.5 \\ 1-x & 0.5\le x\le 1 \end{cases}$	0.25	0.375	0.5

Note

When the fuzzy set is normal, the t-norm T is irrelevant. This is held because a_1=1 and so by lemma 3.1 it is held that

$$MS(A) = N\int_0^1 M(A_\alpha).d\alpha$$

Note that B ⊂ A ⊂ E, so MS(B) ≥ MS(A) ≥ MS(E), and as D ⊂ C then MS(D) ≥ MS(C).

Example 6.4

Many other examples can be generated using different t-norms and negations. Some examples are given using the strong negation N = 1-x^2.

If the t-norm T is the Łukasiewicz t-norm, then

$$MS(C) = W(\alpha_{max}, N(\int_0^{1/2} M(C_\alpha)d\alpha)) = 1/2+N(\int_0^{1/2} M(C_\alpha)d\alpha) - 1 = 1/2 + (3/4)^2 - 1 = 0{,}0625.$$

If T is the product t-norm then

$$MS(C) = Prod(\alpha_{max}, N(\int_0^{1/2} M(C_\alpha)d\alpha)) = 1/2*(3/4)^2 = 0{,}28125.$$

If T is the t-norm minimum then

$$MS(C) = Min(\alpha_{max}, N(\int_0^{1/2} M(C_\alpha)d\alpha)) = Min(1/2, (3/4)^2) = Min(0{,}5, 0{,}5625) = 0{,}5$$

7 Conclusions

A general expression to compute measures of specificity or weak measures of specificity of fuzzy sets under infinite domains is given.

The new expression provides an easy way to compute several measures of specificity of fuzzy sets under infinite domains by choosing a good t-norm, negation and fuzzy measure for each environment or logic.

It is shown that previous examples of measure of specificity under infinite domains in Yager [8] are generalized by the new expression. Some more examples are given.

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