Measures of Specificity of Fuzzy Sets Under T-Indistinguishabilities

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***Abstract*—The concept of measure of specificity of fuzzy sets is extended in this paper to measure of specificity under the knowledge of T-indistinguishabilities. Four axioms of measure of specificity under T-indistinguishabilities are given. An algorithm to compute the inference independent set $\Im$ from a fuzzy set and a T-indistinguishability is used to measure the specificity of fuzzy sets under T-indistinguishabilities satisfying the axioms.**

***Index Terms*—Measure of specificity under similarity, measure of specificity under T-indistinguishability, specificity, T-indistinguishability.**

I. Introduction

The measures of specificity under similarities are introduced as a solution for "the jacket problem" [5]. When some knowledge is given by a similarity, the utility of some non-specific information is increased. For example, if it is known that the temperature is in an interval, it is not itself specific information, but it can be specific and useful in order to choose which jacket to wear today. In general, when our knowledge is increased by a given similarity, then the specificity of fuzzy sets is also increased. This paper studies a first approach of this problem when the existing knowledge is increased by a T-indistinguishability for any t-norm T.

Any α -cut of a similarity relation is a classical equivalence relations, so given a fuzzy set μ on a finite space X and a similarity relation, it is possible to make a partition of X for each value α in $[0,1]$. This fact is used to define measures of specificity under similarities [5]. However, the α-cuts of a T-indistinguishability are not necessarily classical equivalence relations, so new concepts, axioms, definitions and methods are to be introduced to solve this problem.

The axioms of measures of specificity under T-indistinguishabilities given in this paper generalise and extends the previous concept of measures of specificity under similarities.

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An algorithm to compute a measure of specificity under T-indistinguishabilities is given, and it is proved that the output measure verifies the axioms of measures of specificity under T-indistinguishabilities.

II. Preliminaries

Let X be a finite set of elements.

A. *Definition 2.1: Measure of Specificity*

Let $[0,1]^X$ be the set of fuzzy sets on X. A measure of specificity [4], [1], [2] is a function $Sp : [0,1]^X \rightarrow [0,1]$ such that

1) Sp (μ) = 1 if and only if $\mu$is a singleton (μ= {x});
2) Sp $(\emptyset)$ = 0;
3) if $\mu$and η are normal fuzzy sets in X and $\mu \subset \eta$, then Sp $(\mu) \geq$ Sp (η).

The first condition establishes that only crisp sets with just one element (singletons) have the maximum specificity. The second condition assumes that the empty set have minimum specificity. Note that other non-empty fuzzy sets could also have specificity zero. The third condition imposes that the specificity measure of a normal fuzzy set is lower when the membership degree of its elements are increased. A fuzzy set μ on X is normal if there is an element $x \in X$ such that $\mu\ (x)\ = 1$.

The "jacket problem" [5] introduce the concept of specificity of fuzzy sets under similarities. "The weather is over 20° C" is not a specific information, but it is indeed specific in the problem of choosing the right jacket to wear. Some temperature ranges could be similar or indistinguishable in order to make a good decision to choose a jacket to wear.

A fuzzy relation is a T-indistinguishability if it is reflexive, symmetric and T-transitive. A fuzzy relation $R : X \times X \rightarrow [0,1]$ is T-transitive if and only if $T(R(a,b), R(b,c)) \leq R(a,c)$ for all a, b, c in X. When T is the t-norm minimum, then the indistinguishability is a similarity.

The α-cut of a similarity S is a classic equivalence relation denoted S_{α}. Let π_α be the set of equivalence classes of X by the relation S_{α}. Let μ_α/S be the set of equivalence classes of π_α defined as follows: A class $\pi_\alpha(i)$ belongs to μ_α/S if there exists an element x contained in $\pi_\alpha(i)$ and in the α-cut of μ.

The measure of specificity of a fuzzy set μ under a similarity S [5] is computed as

$$S_p(\mu/S) = \int_0^{\alpha_{\max}} \frac{1}{\text{Card}(\mu_\alpha/S)} d\alpha. \tag{1}$$

The measure of specificity under a similarity is maximal when μ_α is contained in one class of S_α for all α in $[0,1]$.

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The given expression is computable when S is a similarity, but it is not valid when S is any T-indistinguishability, because when T is not the minimum t-norm, all the α -cuts of S are not equivalence relations and then μ_α/S is not well defined.

III. Axioms of Measures of Specificity Under a T-Indistinguishability

A. Definition 3.1

Let X be a crisp finite set, let μ be a fuzzy set or a possibility distribution on X, let $S : X \times X \to [0,1]$ be a T-indistinguishability and let Sp be a measure of specificity on fuzzy sets.

$Sp_S(\mu)$ is a **measure of specificity under the T-indistinguishability S** when it verifies the following axioms:

1) $Sp_S(\{x\}) = 1$;
2) $Sp_S(\emptyset) = 0$;
3) $Sp_{Id}(\mu) = Sp(\mu)$;
4) $Sp_S(\mu) \geq Sp(\mu)$.

The first axiom stands that the measures of specificity of a singleton under any T-indistinguishability is always one.

The second axiom extends the second condition of measure of specificity [4].

The empty set does not provide any information, even if a T-indistinguishability is known, so the specificity of the empty set is zero.

The third axiom stands that a measure of specificity of a fuzzy set under the identity relation (Id) is the measure of specificity of the fuzzy set.

The identity relation is a classical equivalence relation that gives a class for each element in X, so the T-indistinguishability does not provide any extra information.

The fourth axiom indicates that when the available information is increased with a T-indistinguishability, then the usability of the information contained in the fuzzy set is always equal or higher. For example, when S is a classical equivalence relation then the election can be done choosing into a lower number of classes. Then a decision is easier and so the measure of specificity should be higher.

IV. Algorithm to Compute an Inference Independent Set

Definition 4.1: The Relation $\succeq$: Let μ be a fuzzy set on a finite set $X = \{x_1, \ldots, x_n\}$, let T be a t-norm and let S be a T-indistinguishability, then the "infers" relation $\succeq$ on $X \times X$ is defined by: $\mathbf{x}_k \succeq x_j$ if and only if $T(\mu(x_k), S(x_k, x_j))) \geq \mu(x_j)$.

If $X_k \succeq X_j$ it is said that X_j is inferred byX_k.

A. Proposition 4.1

The relation $\succeq$ is a classic preorder relation on X.

B. Proof

The $\succeq$ relation is reflexive:

$T(\mu(x_i), S(x_i, x_i))) = T(\mu(x_i), 1) = \mu(x_i), \text{so} x_i \succeq x_i.$

The $\succeq$ relation is transitive.

Let us suppose that $x_i \succeq x_j$ and $x_j \succeq x_k$. Then,

$x_i \succeq x_j, soT(\mu(x_i), S(x_i, x_j)) \geq \mu(x_x).$

$x_j \succeq x_k, soT(\mu(x_j), S(x_j, x_k)) \geq \mu(x_k).$

Hence, $\mu(x_k) \leq T(\mu(x_j), S(x_j, x_k))$

$\leq T(T(\mu(x_i), S(x_i, x_j)), S(x_j, x_k))$ (T is associative)

$= T(\mu(x_i), T(S(x_i, x_j), S(x_j, x_k))$ (S is T-transitive)

$\leq T(\mu(x_i), S(x_i, x_k))$

and so $x_i \succeq x_k$. ■

C. Definition 4.2: Representative Element

Let μ be a fuzzy set on a finite space $X = \{x_1, \ldots, x_n\}$, let T be a t-norm and let S be a T-indistinguishability. An element x_i in X is a representative element when if $x_j \succeq x_i$ then $j = i$.

So, a representative element cannot be inferred from other elements by making fuzzy inference using the t-norm T with the fuzzy set and the T-indistinguishability.

D. Definition 4.3: T-Inference Independent Set

Let μ be a fuzzy set on a finite space $X = \{x_1, \ldots, x_n\}$, let T be a t-norm and let S be a T-indistinguishability.

An inference independent set $\Im$ is defined by a fuzzy set on X such that if x is a representative element then $\Im(x) = \mu(x)$ and, otherwise, $\Im(x) = 0$.

Note that the inference independent set is a fuzzy set $\Im$ on X in which only the representative elements have a positive membership degree.

E. Algorithm

Let μ be a fuzzy set on a finite space $X = \{x_1, \ldots, x_n\}$ sorted by their membership degrees. So $\mu(x_1) \geq \cdots \geq \mu(x_n)$. Let T be a t-norm and let S be a T-indistinguishability on X.

The algorithm to compute the T-inference independent set $\Im$ is given in pseudocode as follows.

$\Im(x_1) := \mu(x_1)$

For $k := 2$ to n do {

  If $\text{Max}_{j=1\cdots k-1} T(\mu(x_j), S(x_j, x_k) \geq \mu(x_k)$

  then $\Im(x_k) := 0$

  else $\Im(x_k) := \mu(x_k)$

}

The algorithm purpose is to get rid in $\Im$ of the elements x_j when it exits $ak, k \neq j$, such that $x_k \succeq x_j$. Those elements are "inferred" by others and then their membership degree in $\Im$ is zero.

F. Example 4.1

Let μ be a fuzzy set on $X = \{x_1, \ldots, x_5\}$ with the following membership degrees : $\mu = 1/x_1, 0.7/x_2, 0.5/x_3, 0.2/x_4, 0/x_5$.

Let S be the following min-indistinguishability:

$$S = \begin{pmatrix} 1 & 1 & 0 & 0.5 & 0.2 \\ 1 & 1 & 0 & 0.5 & 0.2 \\ 0 & 0 & 1 & 0 & 0 \\ 0.5 & 0.5 & 0 & 1 & 0.2 \\ 0.2 & 0.2 & 0 & 0.2 & 1 \end{pmatrix}.$$

The algorithm is applied as follows to compute a min-inference independent set. $\Im(x_1) := \mu(x_1) = 1$.

$k = 2$ computes:

$\text{Max}_{j=1}\{\text{Min}(\mu(x_j), x_2)))\} = \text{Min}(1,1) = 1 \geq \mu(X_2) = 0.7$

Then, x_2 is inferred by x_1, so $\Im(x_2) := 0$.

$k = 3$

$$\text{Max}_{j=1\ldots 2}\{\text{Min}(\mu(x_j), S(x_j, x_3))\} = \text{Max}\{\text{Min}(1,0), \text{Min}(0.7,0))\} = 0 < \mu(x_3) = 0.5.$$

.

Then, x_3 is not inferred by x_1 nor x_2 so $\Im(x_3) := \mu(x_3) = 0.5$.

$k = 4$

$$\text{Max}_{j=1\ldots 3}\{\text{Min}(\mu(x_j), S(x_j, x_4))\} = \text{Max}\{\text{Min}(1, 0.5), \text{Min}(0.7, 0.5), \text{Min}(0.5, 0))\} = 0.5 > \mu(x_4) = 0.2.$$

.

Then, $x_1 \succeq x_4, so \Im(x_4) := 0$.

$k = 5$

$$\text{Max}_{j=1\ldots 4}\{\text{Min}(\mu(x_j), S(x_j, x_5)\} = \text{Max}\{\text{Min}(1, 0.2), \text{Min}(0.7, 0.2), \text{Min}(0.5, 0), \text{Min}\}(0.2, 0.2) = 0.2 > \mu(x_5) = 0.$$

Then, $x_1 \succeq x_5, so \Im(x_5) := 0$.

The fuzzy min-inference independent set $\Im$ is$1/x_1, 0/x_2, 0.5/x_3, 0/x_4, 0/x_5$.

G. Example 4.2

Let T be the t-norm product.

Let μ be a fuzzy set on $X = x_1, \ldots, x_5$ with the following membership degrees: $\mu = 1/x_1, 0.7/x_2, 0.5/x_3, 0.2/x_4, 0/x_5$.

Let S be a prod-indistinguishability represented by

$$S = \begin{pmatrix} 1 & 0.25 & 0 & 0.5 & 0.2 \\ 0.25 & 1 & 0 & 0.5 & 0.2 \\ 0 & 0 & 1 & 0 & 0 \\ 0.5 & 0.5 & 0 & 1 & 0.1 \\ 0.2 & 0.2 & 0 & 0.1 & 1 \end{pmatrix}.$$

Note that S is not a similarity, because $S(x_1, x_2) = 0.25 < \text{Min} S(x_1, x_4), S(x_4, x_2) = \text{Min} 0.5, 0.5 = 0.5$.

The algorithm is applied as follows to compute a prod-inference independent set:

$$\Im(x_1) := \mu(x)_1 = 1$$

$k = 2$

$\text{Max}_{j=1}\{T(\mu(x_j), S(x_j, x_2)\} = T(1, 0.25) = 0.25 < \mu(x_2) = 0.7$

So, x_2 is not inferred by other elements in X and then it is a representative element $\Im(x_2) := \mu(x_2) = 0.7$

$k = 3$

$\text{Max}_{j=1\ldots 2}\{T(\mu(x_j), S(x_j, x_3))\} = \text{Max}\{T(1,0), T(0.7,0)\} = 0 < \mu(x_3) = 0.5$.

Then, x_3 is not inferred by x_1 nor x_2 so $\Im(x_3) := \mu(x_3) = 0.5$.

$k = 4$

$$\text{Max}_{j=1\ldots 3}\{T(\mu(x_j), S(x_j, x_4))\} = \text{Max}\{T(1, 0.5), T(0.7, 0.5), T(0.5, 0)\} = 0.5 > \mu(x_4) = 0.2.$$

Then, $x_1 \succeq x_4, so \Im(x_4) := 0$.

$k = 5$

$$\text{Max}_{j=1\ldots 4}\{T(\mu(x_j), S(x_j, x_5))\} = \text{Max}\{T(1, 0.2), T(0.7, 0.2), T(0.5, 0), T(0.2, 0.1)\} = 0.2 > \mu(x_5) = 0.$$

Then $x_1 \succeq x_5, so \Im(x_5) := 0$.

The fuzzy prod-inference independent set $\Im$ is $1/x_1, 0.7/x_2, 0.5/x_3, 0/x_4, 0/x_5$.

V. Applaying the Algorithm to Compute Measures of Specificity of Fuzzy Sets Under T-Indistinguishabilities

Given a measure of specificity for fuzzy sets, it can be computed a measure of specificity of a fuzzy set under a T-indistinguishability as the measure of specificity of its inference independent set $\Im$.

A. Theorem 5.1

The measure of specificity of μunder a T-indistinguishability $S, Sp_S(\mu)$, computed by $Sp(\Im)$, satisfy the four axioms of measures of specificity under T-indistinguishabilities.

Proof: Let μ be a fuzzy set on a finite space $X = \{x_1, \ldots, x_n\}$ sorted by their membership degree so that $\mu(x_1) \geq \cdots \geq \mu(x_n)$.

Axiom 1:$Sp_S(\{x\}) = 1$.

If μ is a singleton, then $\mu(x_1) = 1$ and $\mu(x_j) = 0$ for all $j \neq 1$.

The steps of the algorithm are computed as follows $\Im(x_1) := \mu(x_1) = 1$.

In further steps, the algorithm runs as follows for $k = 2 \ldots n \text{Max}\{T(1, S(x_1, x_k)), T(0, S(x_2, x_k)), \ldots, T(0, S(x_{k-1}, x_k))\} = T(1, S(x_1, x_k)) = S(x_1, x_k) \geq \mu(x_k) = 0$

So $x_1 \succeq x_k$ for all $k \neq 1$, then $\Im = x_1$ and $Sp_S(\mu) = Sp(\Im) = 1$ because $\Im$ is a singleton.

Axiom 2: $Sp_S(\emptyset) = 0$.

The steps of the algorithm are computed as follows $\Im(x_1) := \mu(x_1) = 0$.

In further steps, the algorithm runs as follows for $k = 2 \dots n$

$$\mathrm{Max}\{T(0, S(x_1, x_k)), T(0, S(x_2, x_k)), \dots, T(0, S(x_{k-1}, x_k))\} = 0 = \mu(x_k) = 0, \mathrm{so} \Im(x_k) := \mu(x_k) = 0.$$

So $\Im = \emptyset$ and $Sp_S(\mu) = Sp(\emptyset) = 0$.

Axiom 3: $Sp_{Id}(\mu) = Sp(\mu)$.

When the T-indistinguishability is the identity, Id, the steps of the algorithm are computed as follows:

$$\Im(x_1) := \mu(x_1).$$

In further steps, the algorithm runs as follows for $k = 2 \dots n$

$$\mathrm{Max}\{T(\mu(x_1), 0), T(\mu(x_2), 0), \dots, T(\mu(x_{k-1}), 0)\} = 0 \leq \mu(x_k)$$

so $\Im(x_k) := \mu(x_k)$ for all k and $Sp_{Id}(\mu) = Sp(\mu)$.

Axiom 4: $S_{ps}(\mu) \geq Sp(\mu)$.

In the first step $\Im(x_1) = \mu(x_1)$. If μ is normal then $\Im$ is also normal. Also, $\Im \subseteq \mu$, and then $Sp_S(\mu) = Sp(\Im) \geq Sp(\mu)$. ■

VI. Examples

A. Example 6.1

Given the min-indistinguishability S in example 4.1, the min-inference independent set of the fuzzy set $\mu = \{1/x_1, 0.7/x_2, 0.5/x_3, 0.2/x_4, 0/x_5\}$ is $\{\Im = 1/x_1, 0/x_2, 0.5/x_3, 0/x_4, 0/x_5\}$. So, the measure of specificity of μ under the min-indistinguishability S is the measure of specificity of the fuzzy set $\Im$.

1) If the given measure of specificity of fuzzy sets is the linear measure [4] defined as

$$Sp(\mu) = \mu(x_1) - \sum_{j=2}^{d} w_j \mu(x_j) \quad (2)$$

with a weight $w_2 = 1$, then the measure of specificity of μ under S is computed as follows: $Sp_S(\mu) = Sp(\Im) = Sp(1/x_1, 0/x_2, 0.5/x_3, 0/x_4, 0/x_5) = 1 - 0.5 = 0.5$. Note that $Sp(\mu) = 1 - 0.7 = 0.3 < Sp_S(\mu)$.

2) If the measure of specificity of fuzzy sets is

$$Sp(\mu) = \mu(x_1) \prod_{j=2}^{d} (1 - w_j \mu(x_j)) \quad (3)$$

with a weight $w_2 = 1$, then the measure of specificity of μunder S is: $Sp_S(\mu) = Sp(\Im) = Sp(1/x_1, 0/x_2, 0.5/x_3, 0/x_4, 0/x_5) = 1 \times (1 - -0,5) = 0.5$.
Note that $Sp(\mu) = 1 \times (1 - -0.7) = 0.3 < Sp_S(\mu)$.

B. Example 6.2

Given the prod-indistinguishability S in Example 4.2, the fuzzy Prod-inference independent set of the fuzzy set $\mu = \{1/x_1, 0.7/x_2, 0.5/x_3, 0.2/x_4, 0/x_5\}$ is $\{\Im = 1/x_1, 0.7/x_2, 0.5/x_3, 0/x_4, 0/x_5\}$.

a) If the measure of specificity of fuzzy sets is the linear measure of specificity (2) with a weight $w_2 = 1$, then the measure of specificity of μ under S is

$$Sp_S(\mu) = Sp(\Im) = Sp(1/x_1, 0.7/x_2, 0.5/x_3, 0/x_4, 0/x_5) = 1 - 0.7 = 0.3.$$

If the chosen weights are $w_2 = 0.5, w_3 = 0.5$, then the measure of specificity of μ under S is $Sp_S(\mu) = Sp(\Im) = Sp(1/x_1, 0.7/x_2, 0.5/x_3, 0/x_4, 0/x_5) = 1 - 0.35 - 0.25 = 0.4$.

b) If the measure of specificity of fuzzy sets is computed by using (3) with weights $w_2 = w_3 = 0.5$, then the measure of specificity of μ under S is $Sp_S(\mu) = Sp(\Im) = Sp(1/x_1, 0.7/x_2, 0.5/x_3, 0/x_4, 0/x_5) = 1 \times (1 - -0.35) \times (1 - -0.25) = 1 \times 0.65 \times 0.75 = 0.4875$.

To solve the problem of making a useful choice of an element in X, the prod-indistinguishability S is telling us that any choice is similar to x_1, x_2 or x_3 and then the choice is easier.

VII. Conclusion

In previous literature on measures of specificity it was not solved the problem of computing measures of specificity under the knowledge of a T-indistinguishability. A simpler problem is solved in [5] under a similarity, whose α-cuts are classical equivalence relations, by integrating the inverse of the cardinality of their classes. However this method is not valid for any T-indistinguishability, because its α-cuts are not necessarily equivalence relations.

This paper solves this problem by giving the axioms of measures of specificity under T-indistinguishabilities and an algorithm to compute the inference independent set $\Im$ from a fuzzy set and a T-indistinguishability. The measures of specificity under T-indistinguishabilities of a fuzzy set $\mu$is computed as the measure of specificity of the fuzzy iinference independent set $\Im$. It is proved that the new proposed measure of specificity under T-indistinguishabilities satisfies the four axioms of measures of specificity under T-indistinguishabilities.

Two examples are provided. The first one (4.1 and 6.1) is using a similarity, and the results are as in [5]. The second example (4.2 and 6.2) is the first approach in literature that uses a T-indistinguishability that is not a similarity.

The "jacket problem" is solved for any T-indistinguishability and a methodological tool is provided to be used in many applications.

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