

# Synchronization effects using a piecewise linear map-based spiking–bursting neuron model

Carlos Aguirre, Doris Campos, Pedro Pascual, Eduardo Serrano\*

*GNB, Escuela Politécnica Superior, Universidad Autónoma de Madrid, 28049 Madrid, Spain*

## Abstract

Models of neurons based on iterative maps allows the simulation of big networks of coupled neurons without loss of biophysical properties such as spiking, bursting or tonic bursting and with an affordable computational effort. These models are built over a phenomenological basis and are mainly implemented by the use of iterative two-dimensional maps that can present neuro-computational properties similar to the usual differential models. A piecewise linear two-dimensional map with one fast and one slow coupled variables is used to model spiking–bursting neural behavior. This map shows oscillations similar to other phenomenological models based on maps that require a higher computational effort. The dynamics of coupled neurons is studied for different coupling strengths and the formation of spatio-temporal patterns of neuronal activity is also explored.

© 2006 Elsevier B.V. All rights reserved.

*Keywords:* Phenomenological neuron model; Iterative maps; Neuron synchronization; Computational complexity

## 1. Introduction

Numerical studies of the collective behavior of ensembles of neurons rely on models of neurons that describe the neuron behavior on the basis of differential equations. Conductance models based on differential equations as the Hodgkin–Huxley (HH) or Hindmarsh–Rose (HR) models require a high computational effort to reproduce neuronal behavior such as spiking or bursting. Such models contain variables of multiple time scales ranging from one millisecond to hundreds of milliseconds to describe slow intrinsic process and require a number of floating point operations that range from 1200 operations in the case of the HH model to 70 operations in the HR model to simulate a single neuron in the network for 1 ms. Recently, some models have been proposed to solve the drawbacks of the differential models [5,7]. These new models are built over a phenomenological basis and are implemented by the use of iterative two-dimensional maps that can present neuro-computational properties similar to the differential models. These models present a low computational effort that makes possible the simulation of big ensembles of

coupled neurons during relatively long periods of time. For example, the sleep and activated states of the thalamocortical system in anesthetized cats have been modeled using a two-dimensional iterative map [8]. The authors demonstrated responses very similar to these found with HH models and experiments.

In the present work a two-dimensional continuous piecewise linear map that models spiking–bursting neural behavior is presented. The map is constructed under phenomenological assumptions and mimics the dynamics of oscillations observed in real neurons. The behavior of coupled neurons is also investigated, in particular the degree of synchronization of a pair of coupled maps is studied for different coupling strengths. The formation of spatio-temporal patterns of neuronal activity in a network is also explored.

## 2. The neuron model and synchronization effects in coupled maps

The modeling of neuronal behavior by means of two-dimensional maps has been object of great interest in the last few years [5,4,7]. These models consider a two-dimensional system that has both a slow and a fast

\*Corresponding author.

*E-mail address:* [eduardo.serrano@uam.es](mailto:eduardo.serrano@uam.es) (E. Serrano).

dynamics. However, the behavior of fast and slow dynamics are clearly correlated. It can be observed that high values of the slow variable correspond with spikes of the fast variable and low values of the slow variable correspond with resting periods of the fast variable.

The previous observation leads us to consider a simplified model implemented as a coupled two-dimensional map  $(y, s)$  where the value of function  $y$  represents the membrane potential. The function  $s$  is a bi-valued function taking the discrete values  $(0, 1)$  and indicates if the value of the membrane potential  $y$  is de-polarizing ( $s = 1$ ) or re-polarizing ( $s = 0$ ) by changing the shape of the function  $y$ . The following equations will be considered for the map

$$y_{n+1} = \begin{cases} \frac{V(s_n)}{B} * y_n & \text{if } 0 \leq y_n < B, \\ (y_n - B) * \frac{K(s_n) - V(s_n)}{C - B} + V(s_n) & \text{if } B \leq y_n < C, \\ (y_n - C) * \frac{T(s_n) - K(s_n)}{D - C} + K(s_n) & \text{otherwise,} \end{cases} \quad (1)$$

$$s_{n+1} = \begin{cases} 0 & \text{if } s_n = 1 \text{ and } y_n > D, \\ 1 & \text{if } s_n = 0 \text{ and } y_n < L \\ & \text{or } (y_n > C - E \text{ and } y_n < C + E), \\ s_n & \text{otherwise,} \end{cases} \quad (2)$$

where  $V(s) = V_0 + s * (V_1 + \sigma)$ ,  $K(s) = K_0 + s * (K_1 + \sigma)$  and  $T(s) = T_0 + s * (T_1 + \sigma)$  and  $B, C, D, E, L, V_1, V_0, K_1, K_0, T_1, T_0$  are non-negative parameters verifying the following conditions:  $L < B < C < D$ ,  $V_0 \leq B$ ,  $V_1 + V_0 \geq B$ ,  $K_0 \leq C$ ,  $K_1 + K_0 \geq C$ ,  $T_0 \leq D$  and  $T_1 + T_0 \geq D$ . The external total input to the neuron is represented by  $\sigma$ .

The parameter  $C$  is the spike threshold. The parameters  $L, E$  and  $D$  are thresholds that force the change of value of the slow variable  $s$  as the fast variable  $y$  crosses one of these thresholds, where  $L$  is the hyperpolarization value and  $D$  is the depolarization value. The other parameters control the dynamics of the membrane potential and they are set to reproduce the behavior of a particular type of neuron. As other map-based model [7], our approach always generates a point on the top of each spike and never misses a spike. A plot of the map is depicted in Fig. 1.

Note that the values of  $V(s)$ ,  $K(s)$  and  $T(s)$  can be computed in advance for both values of the variable  $s$ , and therefore they need not to be computed for each step. This means that for a constant value of the external input  $\sigma$  this map requires at most two floating point operations plus four comparisons per iteration. The descending part of the map  $(y, s)$  ( $s = 0$ ) does not depend on the external input  $\sigma$  presenting so refractory properties of the neuron after spiking.

The temporal behavior of the map (1) for different values of the external input  $\sigma$  is presented in Fig. 2. As can be observed, the map is capable of generating both spiking or bursting series behavior. The parameters for both series

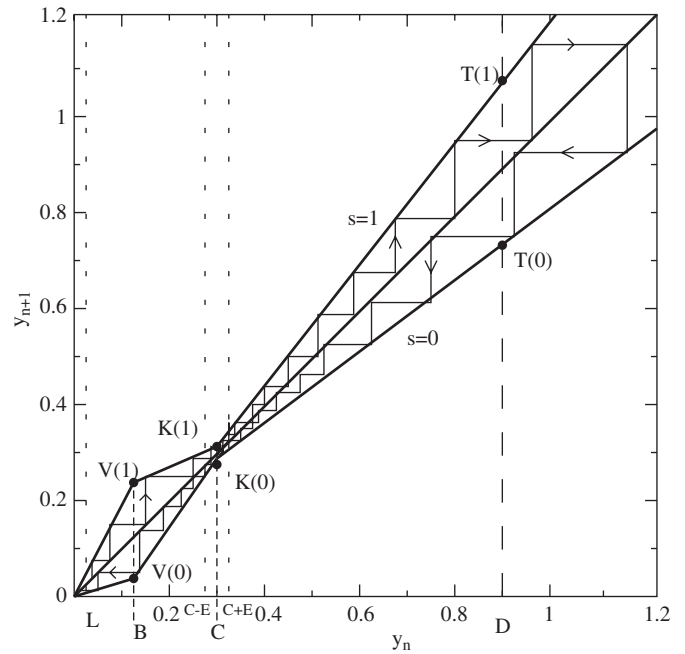


Fig. 1. Graph of the piecewise linear function  $y$ .

are:  $L = 0.01$ ,  $B = 0.15$ ,  $C = 0.3$ ,  $D = 0.9$ ,  $V_0 = 0.14$ ,  $K_0 = 0.29$ ,  $T_0 = 0.75$ ,  $V_1 = 0.01$ ,  $K_1 = 0.02$ ,  $T_1 = 0.4$  and  $S = 0$ . In order to obtain bursting behavior we establish  $E = 0.0055$ . Spiking behavior is obtained by simply establishing  $E = 0$ . Note that the spiking/bursting rate increases with the value of the external input. This behavior is consistent with the results obtained with other phenomenological or differential models (e.g. integrators or class 1 models [4]).

The previous map can be generalized in order to receive synaptic inputs from other neurons in the network. In the generalized mode, a chemical synaptic transmission can be modeled by substituting a constant input  $\sigma$  to neuron  $i$  by the total input at time  $n$

$$\sigma_{n,i} = \sigma_i^e + \frac{1}{\Gamma_i} \sum_{j=1}^N g_{ij} s_{n-1,j} H(y_{n-1,j} - \Theta), \quad (3)$$

where  $\sigma_i^e$  represents both the external input to neuron  $i$  and the action of any current not explicitly captured by the model.  $\Gamma_i$  is the number of neighbors of neuron  $i$ ,  $N$  is the number of neurons in the network,  $y_{n,j}$  and  $s_{n,i}$  represent respectively the value of  $y$  and  $s$  for the neuron  $j$  at time  $n$ . The parameter  $g_{ij}$  is the synaptic coupling coefficient between neuron  $i$  and neuron  $j$  and  $H(x)$  is the usual Heaviside function. The threshold  $\Theta$  has been chosen such that every spike in the single neuron can reach the threshold ( $\Theta = C$ ).

Synchronized neuronal firing has been suggested as particularly relevant for neuronal transmission and coding. The presence of synchronization has been demonstrated in special areas such as the olfactory system [1] or the visual cortex region [3]. Real neurophysiology experiments [2,8]

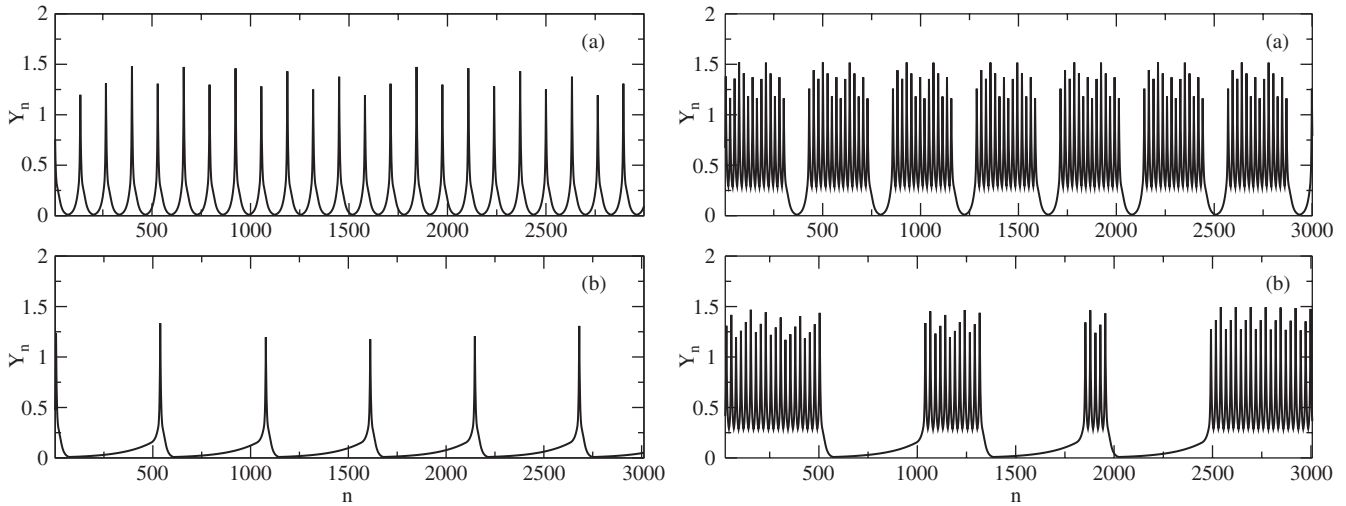


Fig. 2. Spiking (left) and bursting (right) behavior generated by map 1 for (a)  $\sigma = 0.01$  and (b)  $\sigma = 0.001$ . Parameters are described in the text.

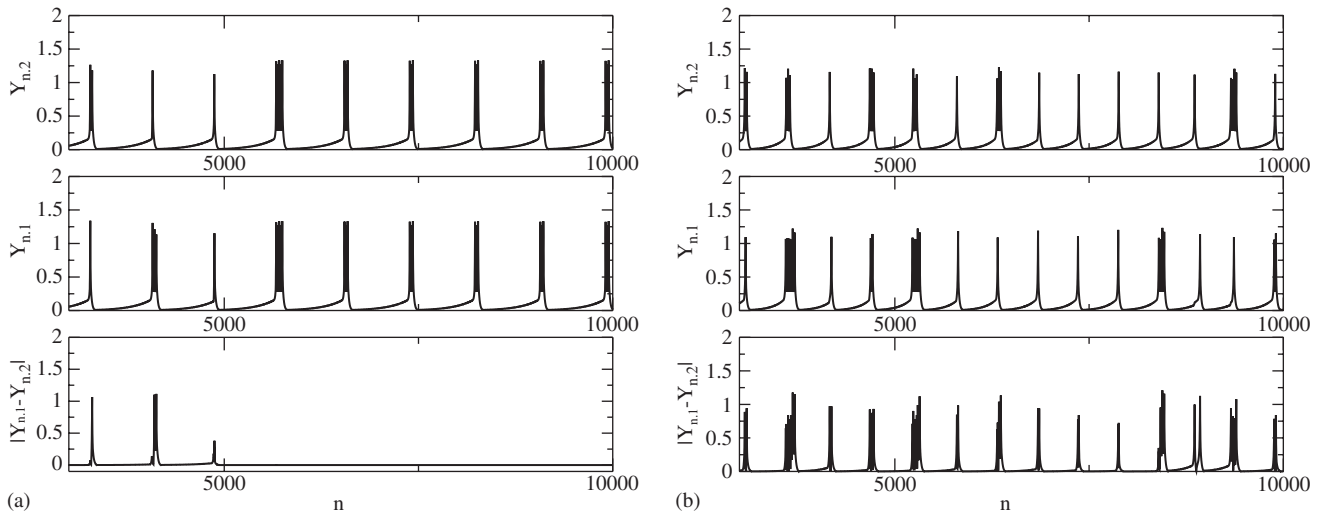


Fig. 3. Synchronization in coupled maps. Parameters are described in the text. (a) Synchronization regime  $g_{1,2} = g_{2,1} = 0.05$  and  $\sigma_e^1 = \sigma_e^2 = 0.001$ . (b) No synchronization  $g_{1,2} = g_{2,1} = 0.005$  and  $\sigma_e^1 = \sigma_e^2 = 0.001$ .

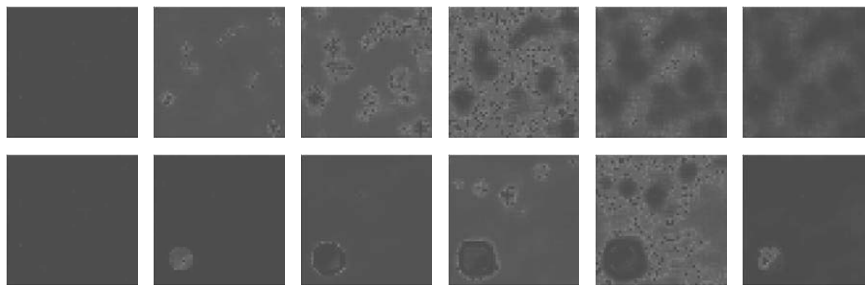


Fig. 4. Sequence of spatio-temporal patterns displayed by the network of  $50 \times 50$  neurons. Neurons with the same color are synchronized. The sequence in the top panel corresponds to a uniform injection of external current  $\sigma_e = 0.005$ . In the bottom panel a set of neurons are stimulated with a higher value of external current  $\sigma_e = 0.01$ .

show that ensembles of coupled neurons can present different regimes of synchronization. These regimes are reproduced both by differential or iterative models [4]. The synchronization phenomena in map (1) can be observed in

Fig. 3 (a) for identical interacting bursting neurons with a symmetric coupling value of  $g_{1,2} = g_{2,1} = 0.05$ . As can be observed in the lower panel of Fig. 3(a) the synchronization of the individual spikes is achieved after a initial transient

period. Note that with a low value of the coupling coefficient  $g_{1,2} = g_{2,1} = 0.005$  the synchronization effect is not clearly obtained as can be seen in Fig. 3(b).

The formation of spatio-temporal patterns of neuron activity in networks such as the inferior olive (IO) has been studied in [9] by means of intensive simulation of a realistic HH model or in vitro by [6]. The role of subcellular and network processes in the genesis of such spatio-temporal patterns in a neural network is not completely understood. Simulations of networks of connected neurons can help to test hypotheses related to the role of these processes. Here we consider a network of identical neurons with a dynamics governed by map 1 and with a synaptic coupling defined by Eq. (1). We built a two-dimensional network of  $50 \times 50$  neurons where each neuron is connected to its 8 closest neighbors. The patterns of network activity generated both under a uniform injection of external current and considering a set of stimulated neurons can be observed in Fig. 4. These patterns of neuronal activity obtained in both cases are very similar to the ones observed both in vivo or [6] with realistic models [9].

### Acknowledgements

We thank the Ministerio de Ciencia y Tecnología (BFI-2003-07276). (PP) and (CA) are partially supported by BFM2002-02359. (PP) and (CA) also receive a partial support by POCTI/MAT/40706/2001. (ES) is partially supported by TIN2004-04363-C03-03.

### References

- [1] M. Bazhenov, M. Stopfer, M. Rabinovich, R. Huerta, H.D.I. Abarbanel, T.J. Sejnowski, G. Laurent, Model of transient oscillatory synchronization in the locust antennal lobe, *Neuron* 30 (2001) 553.
- [2] R.C. Elson, et al., Synchronous behavior of two coupled biological neurons, *Phys. Rev. Lett.* 81 (25) (1998) 5692.
- [3] C.M. Gray, W. Singer, Stimulus-specific neuronal oscillations in orientation columns of cat visual cortex, *Proc. Natl. Acad. Sci. USA* 86 (1989) 1698.
- [4] E.M. Izhikevich, Neural excitability, spiking and bursting, *Internat. J. Bifurc. Chaos* 10 (2000) 1171–1266.
- [5] E.M. Izhikevich, Simple model of spiking neurons, *IEEE Trans. Neural Networks* 68 (2003) 052901.
- [6] E. Leznik, D. Contreras, V. Makarenko, R. Llinas, Markov field analysis of inferior olivary oscillation determined with voltage-dependent dye imaging in vitro, *Soc. Neurosci.* 25 (1999) 1252.
- [7] N.F. Rulkov, Modeling of spiking–bursting neural behavior using two-dimensional map, *Phys. Rev. E* 65 (2002) 041922.
- [8] N.F. Rulkov, I. Timofeev, M. Bazhenov, Oscillations in large-scale cortical networks: map-based model modeling of spiking–bursting neural behavior using two-dimensional map, *J. Comput. Neurosci.* 17 (2004) 203–223.
- [9] P. Varona, C. Aguirre, J.J. Torres, M.I. Rabinovich, H.D.I. Abarbanel, Spatio-temporal patterns of network activity in the inferior olive, *Neurocomputing* 44–46 (2002) 685–690.



**Carlos Aguirre** is a contracted professor at EPS of Universidad Autonoma de Madrid (UAM). He has been researcher from 1990 to 2000 at the Knowledge Engineering Institute. His current research interests are Neural Networks, spiking neuron models and Network topologies.



**Doris Campos** is a Ph.D. candidate at EPS of UAM. Her current research interests are in the signaling activity in neurons and spiking neuron models.



**Pedro Pascual** is a part time associate professor at EPS of UAM. He is also a researcher at Knowledge Engineering Institute at UAM and member of IBM Global Services, Spain. His current research interests are fraud detection in payment systems, complex networks, non-linear systems and anomalous diffusion.



**Eduardo Serrano** is a profesor at EPS of UAM Universidad Autonoma de Madrid. His current research interests are Neural Networks and spiking neuron models.